Capital-Skill Complementarity in Firms and in the Aggregate Economy*

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Abstract

We study capital-skill complementarity in a multi-sector framework featuring firm-specific, multi-factor production functions. We characterize the elasticity of the skill premium to the price of capital equipment in terms of firm-level elasticities of substitution across factors, elasticities of substitution across firms and sectors, and factor intensities. Using French administrative data, we first provide reduced-form evidence that equipment-intensive firms are relatively skill-intensive and that exogenous declines in firm-level equipment prices increase firms' relative demand for skilled labor. We then estimate the micro-level elasticities needed for theory-guided aggregation, providing the first identification of aggregate capital-skill complementarity allowing for arbitrary firm, industry, and aggregate trends in unobserved skill-biased productivity. We find statistically and economically significant aggregate capital-skill complementarity, but this force alone is insufficient to generate the full increase in the relative demand for high-skilled workers observed in the data.

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1 Introduction

Modern technological advances are often embodied in more efficient and less expensive machinery. Following Griliches (1969), a large literature has proposed that such capital-embodied technical advances may be more complementary with high-skilled than low-skilled labor (*capital-skill complementarity*). According to this hypothesis, declining equipment prices increase the relative demand for skilled labor and, therefore, the skill premium. This mechanism is conceptually attractive: the measurable stock of capital equipment proxies for latent skill-biased technical change. It has also proven successful in accounting for the evolution of the skill premium in the United States given particular aggregate elasticities between equipment and high- and low-skilled labor (Krusell et al., 2000).

Despite its virtues, the capital-skill complementarity hypothesis faces a central critique: the aggregate elasticities of substitution that determine the strength of this mechanism are difficult to identify in macro-level time series. In such data, it is challenging to differentiate capital-skill complementarity from other mechanisms that generate skill-biased technical change but are not embodied in capital equipment (Acemoglu, 2002). Hence, whether capital is complementary with skilled labor at the aggregate level and, if it is, the strength of this complementarity remain open questions.

In this paper, we tackle this issue using theory-guided aggregation, obtaining macro-level elasticities of substitution from credibly identified micro-level elasticities. We first develop a theory that characterizes the relevant aggregate elasticities of substitution that determine the degree of capital-skill complementarity in terms of firm-level production elasticities, demand elasticities, and factor intensities. Using data from the universe of French firms, we present reduced-form evidence consistent with capital-skill complementarity within and across firms. We then estimate the micro-level elasticities by leveraging exogenous variation that addresses standard identification concerns, both at the aggregate and at the firm level. We find a statistically and economically significant degree of capital-skill complementarity, but this force alone is insufficient to generate the full increase in the relative demand for high-skilled workers observed in the data. There is a substantial role for skill-augmenting technical change not embodied in capital equipment.

In Section 2, we begin by constructing a multi-sector, general equilibrium model in which firms produce with firm-specific, constant-returns-to-scale production technologies using three factors (high-skilled labor, low-skilled labor, and capital equipment) in fixed aggregate supply. We assume monopolistic competition and a nested CES demand structure across firms and sectors. We show how the response of the real wage of each skill group to a small change in equipment prices depends on aggregate elasticities of substitution between different production factors. These elasticities account for the effect of changes in the price of a given factor on the aggregate demand for another factor. Our primary contribution is characterizing these aggregate elasticities in terms of firm-level elasticities of substitution across factors, factor intensities across firms, and firm-level demand elasticities. We provide a decomposition of the aggregate degree of capital-skill complementarity that highlights three channels through which falling equipment prices can affect the relative demand for skill. First, within firms, equipment may be more complementary with high-skilled than low-skilled labor; if so, this channel raises the relative demand for skill. Second, also within firms, there is a substitution from labor to equipment at fixed output; since the resulting decline in labor demand is stronger for more equipment-intensive firms, this channel reduces the relative demand for skill if these firms are also more skill-intensive. Third, across firms, output reallocates toward firms and sectors that are more equipment intensive; this channel raises the relative demand for skill if the firms and sectors toward which activity reallocates are more skill-intensive.

We generalize our baseline theoretical results along several dimensions that are important for the empirical application. First, we extend our results to allow for exogenous *changes in the supplies of each skill group*, and also for endogenous changes caused by changes in equipment prices when labor supplies are elastic. Second, we extend our environment to allow for firm-specific wedges that create heterogeneity in factor prices across firms. In particular, we show how our baseline results generalize to a case with heterogeneous shocks to the price of equipment, introducing *heterogeneity-adjusted* aggregate elasticities of substitution that incorporate firm-specific exposures to the aggregate equipment-price shock. Moreover, we show how our results generalize to cases in which we consider *composite factors,* e.g., a labor composite that aggregates both skill types, and to cases involving *arbitrarily many factors* where the shock to equipment capital simultaneously shifts the prices of multiple other factors.

We apply our framework to the impact of falling equipment prices on the French skill premium in the late 90s and early 2000s. We exploit a number of administrative firm-level data sources described in Section 3, including firm-level balance sheet data, matched employer-employee data, and transaction-level import and export data. These data sources allow us to directly measure some of the key micro-level variables required for our theory-guided aggregation (including factor intensities across firms) and to estimate the rest (including the elasticities of substitution in production and demand).

Before estimating the full structural model, we present *reduced-form* evidence on the key aggregate patterns and mechanisms (Section 3.3). At the aggregate level, the relative supply of skilled labor rises substantially in France between 1997 and 2007, while the observed skill premium is approximately flat. Hence, relative demand for skill must have increased over the same period. Equipment prices fall substantially over this period, making capital-skill complementarity a natural candidate explanation for the increase in relative demand for skill.

Moving next to the firm level, we show that exogenous declines in equipment prices cause increases in relative skill demand. To do so, we instrument firm-level equipment-price changes using shift-share variation based on firms' initial import exposures across origin countries and subsequent movements in real exchange rates. We also document that, within sectors, equipment intensity positively covaries with relative skill intensity, so that falling equipment prices tend to reallocate activity toward relatively skill-intensive firms. Through the lens of our theory, these reduced-form results suggest that capital is relatively complementary with skilled labor at the aggregate level.

In Section 4 we revisit the above reduced-form facts in the context of our structural model to provide an estimate of the overall contribution of falling equipment prices to the evolution of the skill premium in France. We begin by estimating the key micro-level elasticities needed for our aggregation. First, we estimate the two demand elasticities that characterize substitutability across sectors and across firms within sectors. We construct instruments for firm- and sector-level price changes using the firm-level real-exchange-rate instrument from our reduced-form analysis as well as a version of that instrument constructed at the sector level.

Second, we estimate the elasticities of substitution across different production factors under three alternative restrictions on firm-level production. In our baseline, we use the CRESH family of production functions (Hanoch, 1971), which allows different substitution patterns across the three factors without imposing a nesting structure. To show that the results are not driven by this functional form, we also estimate a specification that imposes constant cross elasticities of substitution directly, which we call CXES, and a nested CES specification in the spirit of Krusell et al. (2000). The CXES specification is particularly useful because it yields a transparent aggregation formula: each component of aggregate capital-skill complementarity in our theoretical decomposition is the product of a data moment that is independent of elasticities and an estimated elasticity. The nested CES specification is useful because it is the functional form used in the canonical aggregate framework in the prior literature. To conduct our estimation, we rely on the firm-level real-exchange-rate instrument, an instrument that shifts firm revenue (related to the world-export-supply instrument introduced by Hummels et al., 2014), and skill-specific wage instruments. The latter leverage differences in the spatial compositions of firm production, the industrial mix of different French regions, and variations in skill intensities and nationwide growth across industries.

Across all three production specifications, we find a moderate degree of capital-skill complementarity at the firm level. In our baseline CRESH specification, the implied average firm-level elasticity of substitution between low-skilled labor and equipment exceeds the corresponding elasticity between high-skilled labor and equipment by about 0.23, in the middle of the range obtained in the reduced-form exercises in Section 3.3. Alternative CRESH specifications as well as specifications imposing CXES and nested CES imply similar magnitudes.

Armed with the estimated elasticities and the observed distribution of factor intensities, we perform our main aggregation exercise in Section 5. Over the 1997-2007 period, we compute a measure of the aggregate degree of capital-skill complementarity—defined as the difference between the aggregate elasticity of substitution between low-skilled labor and equipment and that between high-skilled labor and equipment—of approximately 0.24. Across CRESH specifications, it ranges from 0.17 to 0.24; and across the CXES and nested CES specifications, it ranges from 0.18 to 0.27. These values imply that a 1% decline in the price of equipment raises the observed skill premium by about 0.06–0.09%. Our theoretical decomposition shows that within-firm capital-skill complementarity accounts for most of this aggregate effect, while within-firm equipment-labor substitution and cross-firm reallocation roughly offset each other.

We then quantify the contribution of falling equipment prices to the evolution of the skill premium in France. Between 1997 and 2007, the observed decline in the relative price of equipment generates a model-predicted increase in the skill premium of about 4 log points. However, the relative supply of skilled labor also rises substantially over the same period, pushing the skill premium downward. Combining the observed changes in equipment prices and skill supplies, the model predicts a decline in the skill premium, whereas the observed skill premium is roughly flat. The remaining gap is accounted for by skill-augmenting technical change not embodied in capital equipment.[1]

Finally, we show that our estimated degree of capital-skill complementarity and the elasticity of the skill premium with respect to the price of equipment are substantially smaller than previous estimates obtained from variation in the aggregate time series. For example, our measure of the aggregate degree of capital-skill complementarity is an order

[1]Our quantitative conclusions are robust along several dimensions. They are similar under CRESH, CXES, and nested CES production structures. They are also robust to incorporating heterogeneous equipment-price shocks across firms using our extension with factor-price wedges. Finally, the results are robust to a more flexible three-level demand system.

of magnitude smaller than in Krusell et al. (2000). This is because our strategy does not attribute the aggregate increase in the relative demand for skill *entirely* to capital-skill complementarity.[2]

We view our contribution as twofold. First, in our primary contribution we provide a general theory of measurement: in an economy with many factors, firm-specific and arbitrary production functions, and firm-specific factor-price wedges, we characterize the elasticity of the observed skill premium with respect to the price of capital equipment and the relative supply of skilled labor in terms of firm-level factor intensities and elasticities of production and demand. This framework and decomposition can be applied beyond the French setting we study here. Second, we apply this framework to the French economy. This application has two parts. We first provide reduced-form and structural evidence on the covariances and micro-level elasticities that shape the response of the skill premium to equipment price changes using rich administrative data and plausibly exogenous firm-level variation. We then combine these estimates with our aggregation results to identify aggregate capital-skill complementarity while allowing for arbitrary trends in unobserved skill-biased productivity at the firm, industry, and aggregate levels. The resulting estimates imply that falling equipment prices contributed to the rise in relative skill demand, but that a substantial role remains for skill-augmenting technical change not embodied in equipment capital.

Literature. Our paper contributes to a large literature in labor, international trade, and macroeconomics that studies the implications of technological change for inequality across skill groups. Following Katz and Murphy (1992), many earlier studies measure latent skill-biased technical change as a residual to match the observed evolution of the skill premium given the observed evolution of the aggregate relative supply of skill (see Acemoglu, 2002; Card and DiNardo, 2002, for reviews).[3]

Our approach builds more directly on studies of capital-embodied technical change, which focus on changes in the quality-adjusted price of capital equipment as a well-defined empirical proxy for the technological shock affecting the labor market (Greenwood and Yorukoglu, 1997; Hornstein et al., 2005). This strand of the literature in turn builds on earlier work by Griliches (1969, 1970), who first hypothesized capital-skill com-

[2]This difference in the strength of capital-skill complementarity is not driven by our focus on France: we obtain similar elasticities of the skill premium with respect to the price of capital equipment in France and the U.S. using the approach in Krusell et al. (2000).

[3]A more recent approach in the literature emphasizes the importance of studying the endogenous assignment of production tasks to different skill groups for understanding the effect of technology on inequality (Costinot and Vogel, 2010; Acemoglu and Autor, 2011).

plementarity. In a seminal paper, Krusell et al. (2000) use this idea to provide an account of the rise of the skill premium in the U.S. by estimating a four-factor aggregate production function representation of the U.S. economy using aggregate time-series variation.[4]

A large empirical literature studies the impact of capital equipment on skill intensity at the level of sectors, occupations, or firms, finding strong evidence for the capital-skill complementarity hypothesis at these levels of disaggregation; see, e.g., Autor et al. (1998), Caroli and Van Reenen (2001), Bresnahan et al. (2002), Autor et al. (2003), Bartel et al. (2007), Beaudry et al. (2010), Lewis (2011), Akerman et al. (2015), and Gaggl and Wright (2017).[5] Our contribution relative to this literature is aggregating from micro/firm-level estimates to quantify macro implications.

By providing a framework for explicit aggregation of micro-level firm responses, we build on recent work by Oberfield and Raval (2021), Lashkari et al. (2022), and Baqaee and Farhi (2019). The first two consider two-factor models—thereby abstracting from capital-skill complementarity—with firm heterogeneity under constant-returns-to-scale and variable returns-to-scale technologies, respectively. The latter provides a multi-factor model abstracting from firm heterogeneity.[6] In this spirit, our paper is also closely related to Burstein et al. (2019) and Acemoglu and Restrepo (2022), which quantify the impact of computerization and automation, respectively, on between-group inequality. We differ from these papers in two ways. First, we incorporate firm heterogeneity in our theory and use firm-level data to estimate the elasticities of substitution at the micro level rather than assume their values.[7] Second, we consider the broader notion of equipment capital rather than the more specific notions of computerization or robotization, leaving open whether these specific forms of capital-embodied technical change are more or less complementary with skill than equipment capital.[8]

[4]Duffy et al. (2004) instead use a cross-country panel estimation approach. Unlike these papers, Taniguchi and Yamada (2022) and Caunedo et al. (2023) allow for potential trends in aggregate skill-biased technical change not embodied in capital equipment, with the first leveraging an industry-and-country panel and the second leveraging an occupation panel.

[5]On the other hand, Aghion et al. (2022) find that equipment is neutral and Curtis et al. (2021) find that it is complementary with low-skilled labor.

[6]Unlike these papers, we additionally allow for deviations from perfect factor markets.

[7]The elasticity of substitution at the occupation/task level between labor and computers and between labor and machines is assumed to be one in Burstein et al. (2019) and infinity in Acemoglu and Restrepo (2022), respectively. We note that the relationship between micro-level and macro-level substitution is also widely studied in the context of the effects of trade liberalization on the skill premium (e.g., Parro, 2013; Burstein et al., 2013; Burstein and Vogel, 2017).

[8]We note that some of the mechanisms studied in the literature on computerization and automation/robots such as improvements in what computers can do or changes in the set of tasks technically feasible for robots may appear in our framework as capital-augmenting productivity change rather than capital-skill complementarity.

2 Theory

We begin this section by setting up the baseline environment in Section 2.1. Next, we characterize the response of the skill premium to changes in the price of capital equipment in Section 2.2 and describe how these general results simplify under particular functional form restrictions in Section 2.3. Finally, we discuss a number of extensions of these baseline results in Section 2.4.

2.1 Environment

Factors and the Aggregate Factor Supply. In our baseline analysis, we focus on three factors, indexed by $f \in \mathcal{F} \equiv \{\ell, h, e\}$: low-skilled labor ($\ell$), high-skilled labor ($h$), and capital equipment (e), with perfectly competitive factor markets such that all firms face uniform factor prices. We let $\boldsymbol{W} \equiv \left(W_f\right)_{f\in\mathcal{F}}$ denote the vector of factor prices, normalizing the wage of low-skilled labor to unity, $W_\ell \equiv 1$, such that the *skill premium* W_h/W_ℓ is equal to the high-skilled wage W_h. Furthermore, we let $\overline{\boldsymbol{X}} \equiv \left(\overline{X}_f\right)_{f\in\mathcal{F}}$ denote the vector of aggregate factor supply, which in general is a function of the vector of factor prices $\boldsymbol{W}$.[9] In our baseline analysis, we consider a case where factor supplies are inelastic and exogenously given. However, in an extension we consider the case in which factor supplies are themselves endogenous.

Firm Production. There is a large set $\mathcal{I}$ of firms indexed by $i \in \mathcal{I}$. Firm i produces with a firm-specific constant-returns-to-scale production function $Y_i = G_i\left(\boldsymbol{X}_i\right)$, where $\boldsymbol{X}_i \equiv \left(X_{fi}\right)_{f\in\mathcal{F}}$ indicates the vector of factors of production $f \in \mathcal{F}$ employed by this firm. We assume that the production function $G_i(\cdot)$ is monotone, strictly quasi-concave, and twice continuously differentiable on the interior of its domain. Moreover, we assume Inada-type boundary conditions that ensure firms choose interior solutions for all factors at the equilibria we consider.

Product Demand. Firms compete across a number of sectors indexed by s. We denote by $\mathcal{I}_s \subseteq \mathcal{I}$ the (large) set of firms in sector s. The product market is monopolistically competitive within each sector, and we assume nested CES preferences across sectors.

[9] We assume this function is homogeneous of degree zero. Note, however, that the assumption that this function only depends on factor prices imposes some restriction on the structure of factor supplies, ruling out, e.g., potential nonhomotheticities in factor supplies.

Demand for firm $i \in \mathcal{I}_s$ is given by

$$Y_i = \Phi_i \left(\frac{P_i}{P_s} \right)^{-\varepsilon} \left(\frac{P_s}{P} \right)^{-\eta} Y \tag{1}$$

where ε is the elasticity of substitution across firms within each sector, η is the elasticity of substitution across sectors, Φ_i is a firm-specific demand shifter, P_s is the price index within sector s, and P is the aggregate price index. With slight abuse of notation, we also use P_i to denote the price that firm i charges for its output Y_i. We use Y to denote aggregate production.

2.2 The Skill Premium and a Falling Equipment Price

We now examine the implications of a shock to the price of one factor of production (equipment) on the relative price of two other factors (the skill premium) in general equilibrium. More specifically, we consider a rise in the aggregate supply of equipment $\overline{X}_e$, keeping all other factor supplies fixed. This shock leads to a corresponding fall in the price of equipment. Henceforth, we study the implications of this change in the price of equipment, when measured relative to the low-skilled wage (numeraire), for other factor prices.

2.2.1 Response of the Skill Premium and Aggregate Production

Aggregate Production and Factor Demand. Given our assumptions about product and factor markets, the allocation of inputs across firms is efficient and we can aggregate the production side of our economy.[10] Let $Y = G(\boldsymbol{X})$ denote the resulting aggregate production function, where $\boldsymbol{X} \equiv \sum_{i \in \mathcal{I}} \boldsymbol{X}_i$ denotes the vector of aggregate factor demand across all firms. Correspondingly, given the assumption of CRS, we can define the corresponding total cost function as $\widetilde{C}(\boldsymbol{W}, Y) = Y\, C(\boldsymbol{W})$, where $\boldsymbol{W}$ is the vector of factor prices and $C(\boldsymbol{W})$ is the unit cost function.

The above definitions allow us to write the factor-f aggregate demand $X_f(\boldsymbol{W}, Y) \equiv \partial \widetilde{C} / \partial W_f = Y\, \partial C / \partial W_f$ and aggregate share θ_f in factor payments as

$$\theta_f(\boldsymbol{W}) \equiv \frac{\partial \log C}{\partial \log W_f} = \frac{W_f X_f}{\sum_{f'} W_{f'} X_{f'}}, \tag{2}$$

[10]Note that the assumption of monopolistic competition and the demand system in equation (1) together imply that firms charge constant markups over their marginal costs. As a result, marginal revenue products are equalized across firms.

which we refer to as aggregate factor intensity. Next, we define the aggregate (Allen-Uzawa) elasticity of substitution between factor f and f' (with respect to the price of factor f') as

$$\sigma_{ff'}(\boldsymbol{W}) \equiv \frac{1}{\theta_{f'}} \frac{\partial \log X_f}{\partial \log W_{f'}}, \qquad f \neq f', \tag{3}$$

at fixed aggregate output Y, allowing firm-level outputs to adjust subject to this constraint. The Allen-Uzawa elasticities have two properties that are desirable for our aggregation exercise: pairwise symmetry ($\sigma_{ff'} = \sigma_{f'f}$) and a direct mapping to the curvature (Hessian) of the unit cost function ($\sigma_{ff'} = C \frac{\partial^2 C}{\partial W_f \partial W_{f'}} / \frac{\partial C}{\partial W_f} \frac{\partial C}{\partial W_{f'}}$).[11] Importantly, as we will see below, along with factor intensities, the collection of these cross-factor elasticities is sufficient to fully characterize aggregate responses to small changes in factor prices. For instance, we can write the aggregate own-price elasticity of factor f as a weighted sum of the Allen-Uzawa elasticities of substitution as

$$\frac{\partial \log X_f}{\partial \log W_f} = -\sum_{f' \neq f} \theta_{f'} \sigma_{ff'}. \tag{4}$$

The Skill Premium. According to the definition in equation (3), the direct response of the demand for high-skilled relative to low-skilled labor (henceforth, the relative demand for skill) to a change in the equipment price is given by

$$\frac{\partial \log \left(X_h / X_\ell\right)}{\partial \log W_e} = -\theta_e \left(\sigma_{\ell e} - \sigma_{he}\right). \tag{5}$$

A decline in the price of equipment increases the relative demand for skill if and only if the elasticity of substitution between low-skilled labor and equipment is higher than the elasticity of substitution between high-skilled labor and equipment, that is, $\sigma_{\ell e} - \sigma_{he} > 0$. If this condition is satisfied, we say that the economy exhibits *capital-skill complementarity*.

Next, we study the general equilibrium response of the skill premium, defined as the relative wage of high-skilled workers $W_h / W_\ell \equiv W_h$, to a change in the equipment price. Assuming inelastic labor supplies, wages must respond to equate labor demands with fixed labor supplies. The following proposition characterizes this response.

[11] Blackorby and Russell (1989) emphasize that the Morishima elasticities of substitution, an alternative definition $\sigma^M_{ff'}$ provided in Appendix A.1, have an advantage in terms of interpretation: they measure how a factor-price change affects the demand for one factor relative to another. However, as we show in the appendix, there is a simple mapping between the two sets of elasticities, implying that the collection of Allen-Uzawa elasticities contains the same information about the patterns of cross-factor substitution as the Morishima elasticities.

Proposition 1. *Consider a setting with three factors, $\mathcal{F} \equiv \{\ell, h, e\}$, and with exogenous factor supplies. Assume a small shock to the equipment supply $\overline{X}_e$, holding the supplies of other factors fixed, that leads to a change $d \log W_e$ in the price of equipment. The effect on the skill premium, i.e., the relative prices of high-skilled and low-skilled labor, satisfies*[12]

$$\frac{d\log\left(W_h/W_\ell\right)}{d\log W_e} = -\frac{\theta_e\left(\sigma_{\ell e} - \sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h} + \theta_e \sigma_{he}}, \tag{6}$$

where aggregate factor intensities θ_f and aggregate elasticities of substitution $\sigma_{ff'}$ are defined by equations (2) and (3).

Proof. Since factor supplies and factor demands equalize in equilibrium, $\overline{\boldsymbol{X}} = \boldsymbol{X}$, where $\overline{\boldsymbol{X}}$ denotes aggregate supply and $\boldsymbol{X}$ aggregate demand, and since we have assumed labor supplies remain unchanged, we have

$$0 = \frac{d\log\left(\overline{X}_h/\overline{X}_\ell\right)}{d\log W_e} = \frac{d\log\left(X_h/X_\ell\right)}{d\log W_e} = \frac{\partial \log X_h/X_\ell}{\partial \log W_e} + \left(\frac{\partial \log X_h/X_\ell}{\partial \log W_h}\right)\frac{d\log W_h}{d\log W_e}. \tag{7}$$

The first equality reflects the inelasticity of labor supplies, and the second uses market clearing. Substituting equation (5) in the first term on the right-hand side, and using equations (3) and (4) to write $\frac{\partial \log X_h/X_\ell}{\partial \log W_h} = -(\theta_\ell \sigma_{\ell h} + \theta_e \sigma_{eh} + \theta_h \sigma_{\ell h})$ leads to the desired result. □

When labor is supplied inelastically, Proposition 1 shows that the response of the skill premium depends on two key objects. First, the response depends on the degree of capital-skill complementarity, $\sigma_{\ell e} - \sigma_{he}$. The higher is the gap between these two aggregate substitution elasticities, the more responsive is the relative demand for skill (in the aggregate) to changes in the price of equipment.

Second, the response also depends on a weighted average of the aggregate elasticity of substitution between high- and low-skilled labor, $\sigma_{h\ell}$, and between high-skilled labor and equipment, σ_{he}. When a falling equipment price encourages a rise in skill demand due to capital-skill complementarity, the skill premium has to increase to re-establish labor market clearing. When the two labor types are more substitutable (high $\sigma_{h\ell}$), equilibrating the labor market requires a smaller increase in the skill premium.

[12] As shown in Appendix A.1, when expressed in terms of the aggregate Morishima elasticities of substitution, the response can be expressed equivalently as

$$\frac{d\log\left(W_h/W_\ell\right)}{d\log W_e} = -\frac{\sigma^M_{\ell e} - \sigma^M_{he}}{\sigma^M_{\ell h}}.$$

We can further derive the implications of a falling equipment price for the *real* wages of the two skill groups. Since markups are constant and the low-skilled wage is the numeraire, we find[13]

$$\begin{aligned} \frac{d\log\left(W_h/P\right)}{d\log W_e} &= -\theta_e\left(1+\frac{\left(1-\theta_h\right)\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}\right), \\ \frac{d\log\left(W_\ell/P\right)}{d\log W_e} &= -\theta_e\left(1-\frac{\theta_h\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}\right) \end{aligned} \tag{8}$$

If equipment is equally complementary with high- and low-skilled labor, $\sigma_{\ell e}=\sigma_{he}$, then a reduction in the price of equipment raises the real wage of high- and low-skilled labor proportionately. On the other hand, in the presence of capital-skill complementarity, $\sigma_{\ell e}>\sigma_{he}$, a reduction in the equipment price necessarily raises the real wage of high-skilled labor, but may either raise or lower the real wage of low-skilled labor. Specifically, low-skilled labor experiences a loss in real terms if the degree of capital-skill complementarity is sufficiently large, in the sense that[14]

$$\sigma_{\ell e}-\sigma_{he}>\frac{1}{\theta_h}\left[\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}\right]. \tag{9}$$

Thus, in our general setup, technological progress embodied in capital equipment, in the form of technologies reducing the price of equipment, may reduce the real wage of low-skilled labor, a point to which we return in Section 2.3.[15]

While Proposition 1 is stated in the three-factor environment, the same logic applies with an arbitrary number of factors. In that more general case, the collection of aggregate substitution elasticities and factor intensities determines the responses of all other relative factor prices to the shock in the price of one factor through a linear system. Appendix A.3.4 provides the corresponding vector formulation, working in relative prices because factor demand is homogeneous of degree zero in factor prices. Section 2.4 discusses this extension, along with two other extensions that account for variations in the supply of labor by the two skill groups and potential heterogeneity in factor prices across firms. The core insight of Proposition 1 remains in all these extensions: the degree of capital-skill complementarity and the degree of substitutability of the two skill groups play key

[13]We have $\frac{d\log P}{d\log W_e}=\frac{d\log C}{d\log W_e}=\frac{\partial\log C}{\partial\log W_e}+\frac{\partial\log C}{\partial\log W_h}\frac{d\log W_h}{d\log W_e}$, where the first equality follows from constant markups and the definition of C as the unit cost function. Shephard's lemma implies that $\frac{\partial\log C}{\partial\log W_f}=\theta_f$.

[14]See the derivation in Appendix A.4.1 on page A16.

[15]Symmetric results apply if equipment is relatively more complementary with low-skilled than high-skilled labor, in which case a reduction in the price of equipment raises the real wage of low-skilled labor but may either increase or decrease the real wage of high-skilled labor.

roles in shaping the response of the skill premium to a falling equipment price.

2.2.2 From Firm to Aggregate Production

Firm Factor Demand. Here, we characterize firm-level factor demand symmetrically to aggregate factor demand. As before, we define the total cost function corresponding to the production function $G_i(\cdot)$ as $\widetilde{C}_i(\boldsymbol{W}, Y_i) = Y_i C_i(\boldsymbol{W})$ and the factor-f firm-level demand $X_{fi}(\boldsymbol{W}, Y_i) \equiv \partial \widetilde{C}_i / \partial W_f = Y_i \partial C_i / \partial W_f$ and its share in firm-level factor payments θ_{fi} in factor payments (factor intensity) as

$$\theta_{fi}(\boldsymbol{W}) \equiv \frac{\partial \log C_i}{\partial \log W_f} = \frac{W_f X_{fi}}{\sum_{f'} W_{f'} X_{f'i}}. \tag{10}$$

Similarly, the elasticity of substitution between factor f and f' (with respect to the price of factor f') for firm i is

$$\sigma_{ff',i}(\boldsymbol{W}) \equiv \frac{1}{\theta_{f'i}} \frac{\partial \log X_{fi}}{\partial \log W_{f'}}, \qquad f \neq f', \tag{11}$$

at fixed firm-level output, Y_i, and the own price elasticity of factor f is

$$\frac{\partial \log X_{fi}}{\partial \log W_f} = -\sum_{f' \neq f} \theta_{f'i} \sigma_{ff',i}. \tag{12}$$

Aggregating Firm Factor Demand. Let Λ_i denote firm i's share of aggregate factor payments (costs), defined as

$$\Lambda_i \equiv \frac{Y_i C_i}{YC} = \frac{\sum_f W_f X_{fi}}{\sum_f W_f X_f}, \tag{13}$$

and, with a slight abuse of notation, let $\Lambda_s \equiv \sum_{i \in \mathcal{I}_s} \Lambda_i$ denote the corresponding share across all firms in sector s. For notational convenience, define firm and sector factor intensities relative to their aggregate counterparts as

$$\widehat{\theta}_{fi} \equiv \frac{\theta_{fi}}{\theta_f}, \qquad \widehat{\theta}_{fs} \equiv \frac{\theta_{fs}}{\theta_f}. \tag{14}$$

Using these definitions, we next decompose the aggregate substitution elasticities in terms of firm-level elasticities. The logic behind this decomposition is straightforward. A change in the price of a given factor of production leads to two distinct responses: a within-firm substitution governed by the firm-level elasticities $\sigma_{ff',i}$, and additional reallocation terms, because factor-price changes affect firm marginal costs and therefore shift output across firms within and across sectors. Aggregating these responses using the cost-share weights defined above yields the aggregate elasticities of substitution, as stated in the following proposition.

Proposition 2. *The aggregate elasticity of substitution $\sigma_{ff'}$ of aggregate factor demand defined by equation (3) is given by*

$$\sigma_{ff'} = \mathbb{E}_i\left[\widehat{\theta}_{fi}\,\widehat{\theta}_{f'i}\,\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\widehat{\theta}_{fi}, \widehat{\theta}_{f'i}\right)\right] - \eta\,\mathbb{C}_s\left(\widehat{\theta}_{fs}, \widehat{\theta}_{f's}\right) \tag{15}$$

where $\widehat{\theta}_{fi}$ and $\widehat{\theta}_{fs}$ are the relative firm and sector factor intensities defined in equation (14), $\sigma_{ff',i}$ are the firm-level production elasticities defined in equation (11), the firm-level expectation operator $\mathbb{E}_i[\cdot]$ is defined with respect to the share of firms in aggregate costs (factor payments) Λ_i defined by equation (13), the within-sector firm-level covariance operator $\mathbb{C}_{i|s}(\cdot,\cdot)$ is defined with respect to the share of firms in their respective sectors $\Lambda_{i|s} \equiv \Lambda_i/\Lambda_s$, and the sector-level expectation and covariance operators $\mathbb{E}_s[\cdot]$ and $\mathbb{C}_s(\cdot,\cdot)$ are defined with respect to the share of sectors in aggregate costs Λ_s.

Proof. See Appendix A.4 on page A15. □

Equation (15) expresses the aggregate elasticity of substitution as a function of firm-level elasticities of substitution and factor intensities, elasticities of demand, and the distribution of costs across firms. The first term on the right-hand side of equation (15) is a weighted average across firms of firm-level elasticities of substitution, scaled by relative factor intensities. The second term accounts for the effect of cross-firm reallocation within sectors on the aggregate elasticity of substitution. If factor intensities for the two factors f and f' positively covary across firms within each sector, a rise in the price of either of the two factors lowers the aggregate demand for the other factor by shifting production away from firms intensive in both factors. The magnitude of this effect is governed by the demand-side, within-sector elasticity of substitution ε. The last term on the right-hand side of equation (15) captures a similar force that operates across sectors, governed in magnitude by the demand-side, cross-sector elasticity of substitution η.

From Micro to Macro Capital-Skill Complementarity. We can use equation (15) to compute the three aggregate elasticities, $\sigma_{\ell e}$, σ_{he}, and $\sigma_{\ell h}$, featured in the results of Proposition 1. For compactness, let $\widetilde{\theta}_{ni} \equiv \frac{1}{2}\widehat{\theta}_{\ell i} + \frac{1}{2}\widehat{\theta}_{hi}$ denote average relative labor intensity and let $\widetilde{\sigma}_{ne,i} \equiv \frac{1}{2}\sigma_{\ell e,i} + \frac{1}{2}\sigma_{he,i}$ denote the average labor-equipment elasticity of substitution at the firm level. Accordingly, for the degree of capital-skill complementarity we find[16]

$$
\begin{aligned}
\sigma_{\ell e} - \sigma_{he} = & \overbrace{\mathbb{E}_i^e\left[\widetilde{\theta}_{ni}\right] \times \mathbb{E}_i^e\left[\sigma_{\ell e,i} - \sigma_{he,i}\right] + \mathbb{C}_i^e\left(\widetilde{\theta}_{ni},\, \sigma_{\ell e,i} - \sigma_{he,i}\right)}^{\text{within-firm capital-skill complementarity}} \\
& - \overbrace{\left[\mathbb{C}_i\left(\widehat{\theta}_{hi} - \widehat{\theta}_{\ell i}, \widehat{\theta}_{ei}\right) \times \mathbb{E}_i^e\left[\widetilde{\sigma}_{ne,i}\right] + \mathbb{C}_i^e\left(\widehat{\theta}_{hi} - \widehat{\theta}_{\ell i},\, \widetilde{\sigma}_{ne,i}\right)\right]}^{\text{within-firm equipment-labor substitution}} \\
& + \underbrace{\varepsilon \times \mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\widehat{\theta}_{hi} - \widehat{\theta}_{\ell i}, \widehat{\theta}_{ei}\right)\right]}_{\text{cross-firm (within-sector) reallocation}} + \underbrace{\eta \times \mathbb{C}_s\left(\widehat{\theta}_{hs} - \widehat{\theta}_{\ell s}, \widehat{\theta}_{es}\right)}_{\text{cross-sector reallocation}}.
\end{aligned} \tag{16}
$$

where the firm-level covariance operator $\mathbb{C}_i(\cdot,\cdot)$ is defined with respect to the share of firms in aggregate factor payments Λ_i, and the expectation and covariance operators $\mathbb{E}_i^e[\cdot]$ and $\mathbb{C}_i^e(\cdot,\cdot)$ are defined with respect to the share of firms in aggregate equipment demand $\Lambda_i\widehat{\theta}_{ei}$.

The two terms under the *within-firm capital-skill complementarity* brace in equation (16) account for the direct contribution of firm-level capital-skill complementarity to the aggregate. The first term captures the equipment-weighted mean of firm-level capital-skill complementarity, scaled by firms' average labor intensity. The second term captures the corresponding covariance between average labor intensity and firm-level capital-skill complementarity.

The two terms under the *within-firm equipment-labor substitution* brace account for substitution between equipment and labor at fixed output for each firm. When the equipment price falls, more equipment-intensive firms substitute more strongly toward equipment. If these firms are also more intensive in skilled compared to unskilled labor, this component lowers aggregate relative demand for skill.

The final two terms account for output reallocation across firms within sectors and across sectors, respectively. When more equipment-intensive firms or sectors are also more intensive in skilled compared to unskilled labor, a fall in the equipment price shifts output toward those producers and raises aggregate relative demand for skill through these reallocation terms.

2.3 Functional Forms for Firm Production

In this section we discuss our general results in the context of specific functional form restrictions on firm-level production technologies. We begin with a firm-level production

[16] See the derivation in Appendix A.4.1 on page A17.

function that abstracts from firm-level, but not aggregate, capital-skill complementarity, before turning to less restrictive settings.

Example 1 (CES Production Function)**.** Consider a firm-level production function $Y_i = G_i(\boldsymbol{X}_i)$ characterized by

$$Y_i = \left(\sum_f (Z_{fi}X_{fi})^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}$$

where Z_{fi} denotes firm-i, factor-f-augmenting productivity. The elasticities of firm-level technology are given by $\sigma_{ff',i} \equiv \zeta$, which implies that there is no capital-skill complementarity at the firm level and the first term in equation (16) is zero. Nevertheless, the economy may feature capital-skill complementarity at the aggregate level. In particular, assuming a single-sector model (equivalently, $\eta = \varepsilon$), equation (16) simplifies further to

$$\sigma_{\ell e} - \sigma_{he} = (\varepsilon - \zeta)\,\mathbb{C}_i\left(\widehat{\theta}_{hi} - \widehat{\theta}_{\ell i}, \widehat{\theta}_{ei}\right).$$

When the elasticity of substitution across products exceeds that across inputs, $\varepsilon > \zeta$, and when equipment intensity covaries positively with the relative demand for skill across firms, we find capital-skill complementarity in the aggregate even though firm-level production does not feature such complementarity. This result generalizes the logic of Houthakker (1955) and Oberfield and Raval (2021) from a two-factor to a multi-factor setting.

Example 2 (Nested CES Production Function à la Krusell et al., 2000)**.** Consider a firm-level production function $Y_i = G_i(\boldsymbol{X}_i)$ characterized by the following nested structure

$$Y_i = \left((Z_{\ell i}X_{\ell i})^{\frac{\varsigma-1}{\varsigma}} + X_{ci}^{\frac{\varsigma-1}{\varsigma}}\right)^{\frac{\varsigma}{\varsigma-1}}, \qquad X_{ci} \equiv \left((Z_{hi}X_{hi})^{\frac{\rho-1}{\rho}} + (Z_{ei}X_{ei})^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{17}$$

where X_{ci} is a composite factor that combines high-skilled labor and equipment capital.

As shown in Appendix A.2.1, the functional form in Example 2, which we will henceforth refer to interchangeably as KORV, implies the following elasticities of substitution

$$\sigma_{he,i} = \varsigma - (\varsigma - \rho)\frac{1}{\theta_{ei} + \theta_{hi}}, \qquad \sigma_{\ell e,i} = \sigma_{\ell h,i} = \varsigma. \tag{18}$$

This functional form assumption implies that the elasticity of substitution between low-skilled labor and equipment, $\sigma_{\ell e,i}$, equals the elasticity of substitution between low-skilled and high-skilled labor, $\sigma_{\ell h,i}$, within firm i. The assumption that the CES elasticities ς

and ρ are common across firms additionally implies that these cross-factor elasticities of substitution are common across firms, $\sigma_{\ell e,i} = \sigma_{\ell h,i} = \varsigma$. Choosing $\varsigma = \rho$, we recover the standard CES production function, which implies $\sigma_{\ell e,i} = \sigma_{he,i}$ and does not allow for firm-level capital-skill complementarity, as described above in Example 1. More generally, the degree of firm-level capital-skill complementarity is given by $\sigma_{\ell e,i} - \sigma_{he,i} = (\varsigma - \rho)/(\theta_{e,i} + \theta_{h,i})$, and the production function exhibits capital-skill complementarity if and only if $\varsigma > \rho$. The nested CES specification imposes strong restrictions on the feasible strength of capital-skill complementarity. To see this most clearly, consider an environment with symmetric firms, in which case we can express the response of the real wage of low–skill workers in equation (8) as[17]

$$\frac{d\log\left(W_\ell/P\right)}{d\log W_e} = -\theta_e \frac{\left(\theta_e + \theta_h\right)\rho}{\theta_h \varsigma + \theta_e \rho} < 0. \tag{19}$$

The nested CES specification implies that a falling equipment price necessarily increases the real wage of low-skilled workers.

Example 3 (CRESH Production Function à la Hanoch, 1971)**.** Consider a firm-level production function $Y_i = G_i\left(\boldsymbol{X}_i\right)$, implicitly defined through the following constraint on unit factor requirements:

$$\sum_{f \in \{\ell,h,e\}} \delta_f \left(\frac{Z_{fi} X_{fi}}{Y_i}\right)^{\frac{\sigma_f - 1}{\sigma_f}} = 1, \tag{20}$$

where $\sigma_f > 0$ with $\sigma_f \neq 1$ control the elasticities of substitution, where $Z_{fi} > 0$ shape the factor-augmenting productivity, and where $\delta_f \equiv \operatorname{sgn}(\sigma_f - 1)$.[18]

Hanoch (1971) introduced the class of production functions and referred to them as CRESH: constant ratios of elasticities of substitution with homotheticity. Equation (20) is a homothetic generalization of the standard CES production function allowing for different degrees of substitutability among different factor inputs. The CES production function is nested in this specification for the case of constant parameters $\sigma_f \equiv \zeta$ across all factors f. As shown in Appendix A.2.2, the functional form in Example 3 implies that firm-level elasticities of substitution are given by

$$\sigma_{ff',i} = \frac{\sigma_f \sigma_{f'}}{\overline{\sigma}_i}, \tag{21}$$

[17] See the derivation in Appendix A.4.1 on page A17.

[18] The definition above assumes that at least one $\sigma_f > 1$. In the case where $\sigma_f < 1$ for all f, we instead define $\delta_f \equiv 1$ for all f. Our assumptions satisfy the conditions stated in Hanoch (1971) to ensure that the value of the production function is unique, monotonically increasing in all factors X_{fi}, and quasi-concave globally if $X_{fi} > 0$ for all f.

where we have defined $\overline{\sigma}_i \equiv \sum_f \theta_{fi}\sigma_f$ as the firm-specific cost-share weighted average of σ_f across factors. Thus, we find that the degree of capital-skill complementarity at the firm level is given by $\sigma_{\ell e,i} - \sigma_{he,i} = \sigma_e \left(\sigma_\ell - \sigma_h\right) / \overline{\sigma}_i$ and production exhibits capital-skill complementarity if and only if $\sigma_\ell > \sigma_h$. Parameters σ_h and σ_ℓ characterize the relative substitutability of high- and low-skilled workers with equipment.

Unlike the nested CES specification, the strength of capital-skill complementarity is not restricted: it can be sufficiently large as to allow a falling equipment price to reduce the real wage of low-skilled labor. As above, consider the case of symmetric firms. Under the specification in Example 3, low-skilled labor experiences a loss in real terms in response to a decline in the equipment price if[19]

$$\frac{\sigma_\ell}{\sigma_h} > 1 + \frac{\theta_e}{\theta_h} + \left(\frac{1-\theta_e}{\theta_h}\right)\frac{\sigma_\ell}{\sigma_e}. \tag{22}$$

Example 4 (Constant Cross Elasticities of Substitution)**.** Consider a restriction imposed directly on firm-level elasticities of substitution between factors such that cross elasticities are common across firms, $\sigma_{ff',i} \equiv \zeta_{ff'}$ for all firms i, where $\zeta_{ff'}$ is the common firm-level elasticity between factors f and f'.

The CXES restriction has a particularly nice aggregation property. The second terms in both the within-firm capital-skill complementarity channel and in the within-firm equipment-labor substitution channels in equation (16) are both zero. Hence, each of the four components of the aggregate strength of capital-skill complementarity simplifies into the product of a single data moment that can be constructed independently of all elasticities and a single elasticity.

This approach has clean aggregation results, but suffers from the standard critique waged by the so-called Uzawa-McFadden impossibility results (Uzawa, 1962; McFadden, 1963), which show that empirical specifications of factor demand featuring constant and heterogeneous elasticities of substitution are often incompatible with well-defined production functions.[20]

[19] See Appendix A.4.1 on page A16 for the derivation.

[20] More specifically, their results establish the conditions that any production function needs to satisfy in order to exhibit heterogeneous and fixed cross-factor elasticities of substitution. For instance, under the Allen-Uzawa definition of the substitution elasticities considered here, Uzawa (1962) shows that the only production function compatible with fixed cross-factor substitution elasticities can be written as a Cobb-Douglas aggregate of CES nests, implying constant substitution elasticities within nests and unitary substitution elasticities across nests.

2.4 Theoretical Extensions

In this section, we discuss a number of theoretical extensions to our benchmark setup in Section 2.1.

2.4.1 Shifts in Relative Labor Supply

So far, we studied how the skill premium responds to a change in the price of equipment only. As is well-known (Katz and Murphy, 1992; Goldin and Katz, 2007), the relative supply of skill in the U.S. and across most other economies has been increasing for several decades. The effects of these trends in skill supply can easily be integrated into our framework. First, we can use our framework to account for the contribution of exogenous changes in the relative supply of skill on the skill premium. As another extension, we can further consider an environment in which the relative supply of skill responds endogenously to changes in the skill premium, with an elasticity $\xi > 0$. In this case, changes in the price of equipment impact the skill premium both directly (as in our baseline) and indirectly through their effect on skill supply. Our key result holds in the sense that equation (5) still characterizes the extent of capital-skill complementarity and is the key determinant of the response of the skill premium to changes in the price of equipment capital. Appendix A.3.1 formalizes these arguments and characterizes the evolution of the skill premium in response to a change in the price of equipment in this extended setting.

2.4.2 Factor-Price Heterogeneity

In our benchmark setting, we assume perfectly competitive factor markets in which all firms face common factor prices. However, our empirical application relies on variation in observed prices of equipment capital across firms. We can account for such cross-firm heterogeneity in equipment prices as a generalization of our framework by assuming *exogenous* wedges in equipment prices across firms, in which firm i faces equipment price $W_{ei} = T_{ei} W_e$ with T_{ei} denoting an exogenous firm-specific wedge and W_e a shadow price of equipment. Such heterogeneity in wedges may stem from cross-firm differences in the prices of imported equipment, which we document in our data, as driven by differences in the composition of imported equipment, their origins, and the corresponding import costs.[21]

We can further generalize our results to cases featuring heterogeneity in factor prices across all factors, including skilled and unskilled labor. In this case, one needs to make a

[21] Appendix A.6 formalizes this specific micro-foundation for variations in exogenous price wedges across firms.

distinction between the *observed skill premium*, which accounts for observed heterogeneity in wages across firms, and the theoretical notion of the *shadow skill premium* that ensures the clearance of skill markets in general equilibrium. Appendix A.3.2 presents this generalized environment featuring factor price heterogeneity and characterizes the equilibrium responses of the observed and the shadow skill premium to a fall in equipment prices.

2.4.3 Composite Factors of Production

While we make a distinction between high- and low-skilled labor, our framework can also be used to derive predictions about the response of labor demand and income share to changes in the price of equipment, defining labor as a composite factor n that aggregates both skill groups, such that $X_n \equiv X_\ell + X_h$. Appendix A.3.3 derives the aggregate- and firm-level elasticities of substitution for this composite factor in terms of the substitution elasticities defined for the underlying, more disaggregated factors of production.

2.4.4 Additional Factors of Production

We can generalize our framework to accommodate additional factors of production beyond the three considered in Section 2.1. Our definitions for firm- and aggregate-level elasticities of substitution remain intact in this case. However, now we must account for the response in the prices of the other factors of production to the change in the price of equipment in computing the response of the skill premium. Proposition A.6 in Appendix A.3.4 generalizes Proposition 1 and characterizes the responses of all factor prices, including the skill premium, to a change in the price of equipment in terms of aggregate factor intensities and aggregate elasticities of substitution.

3 Data and Reduced-Form Evidence

3.1 Data Sources

We exploit a number of French administrative data sources on firms, workers, and firm-level export and import transactions. See Section B in the Online Appendix for a full description of these sources and additional details on the variables used.

Balance sheet and administrative information are retrieved from BRN (*Bénéfices Réels Normaux*), a dataset jointly administered by the *Direction Générale des Finances Publiques* (DGFiP) and the French *Institut National de la Statistique et des Études Économiques* (INSEE). BRN covers all French enterprises filing under the normal tax regime over the period

1993-2009, which provides a sufficiently long pre-sample period to construct equipment capital stock series.[22] Although firms under the normal regime are only 24% of all French firms, they account for 94.3% of total gross output and for over 90% of the aggregate value of trade flows in customs records, so this restriction retains nearly all economic activity.[23] This data source has been widely used in the literature (e.g., Eaton et al., 2011) and, crucially for us, it contains the detailed breakdown of firm capital by asset type.

Firm-level international transaction data are provided by the French *Directorate-General of Customs and Indirect Taxes* (DGDDI), which provides information on the annual value of imports and exports by country of origin/destination at the level of 8-digit CN product codes for all firms involved in international transactions (with simplified declarations below certain thresholds). We make use of the data over the 1994–2007 period and our final dataset describes the international trade flows of French firms concerning over 5,000 products from all origins and to all destinations.

As we will discuss in detail in Section 3.2 below, firm-level employment and wage bill data are computed by aggregating worker-level data on hours worked and wages. These data, together with worker characteristics, are retrieved from the DADS (*Déclarations Annuelles de Données Sociales*) Poste, a matched employer-employee dataset provided by INSEE that covers the whole population of French workers.

We construct two different samples from the datasets described above. We use the "aggregation sample", which includes all firm-year observations, for the quantification exercise in Section 5, and a smaller "estimation sample" for the estimation of firm-level production elasticities in Section 4. Appendix B.4 provides further details on the construction of both samples, and Table B.1 reports their summary statistics.

We use sectoral data on employment, investment, and capital by asset type from the Annual National Accounts compiled by Eurostat. Industry-level data on depreciation rates by asset type come from the EU-KLEMS Database.

3.2 Variable Definitions

In this section we provide a brief overview of how we compute the main variables of interest used in the analysis. See Appendix C for further details.

In our model, firm revenue equals value added. Annual value added for each firm i is

[22] Firms with revenues above a certain threshold must be affiliated with the BRN regime, while firms below the threshold are not required but always have the option to opt for it (instead of the simplified regime RSI). In 2007, the thresholds were 763,000 euros if a firm operates in trade or real estate sectors and 230,000 euros otherwise.

[23] See INSEE (2004) for further information about the representativeness of BRN.

obtained from BRN as gross output less intermediate expenditures, where gross output is defined as the sum of sales and changes in inventories while intermediates include all the purchases of the firm.

The DADS data do not provide information on the education of workers, but include their detailed occupational codes. Following prior work (e.g., Caliendo et al., 2015; Carluccio et al., 2015), we assign workers to two skill groups based on their occupation, using occupational classifications in French social security data (CS - catégories socioprofessionnelles). High-skill workers are those employed as Business Owners and Executives, Managers and Higher-level Intellectual Occupations, and Intermediate Occupations. Low-skill workers are employed as Clerks and Blue-collar Workers.

In the model, all workers in a skill group are identical; but in practice, workers within the same group vary in observable and unobservable characteristics. To measure five-year, firm-and skill-specific wage changes in the data (Δw_{hit} and $\Delta w_{\ell it}$), we correct changes in observed wages both for changes in returns to worker observable characteristics and for changes in worker composition (along observable and unobservable characteristics). To do so, we estimate a national Mincerian regression of log wage changes between each consecutive time period $t-1$ and t. In this regression, we control for worker observables (gender, full-time status, 2-digit CS category), including additionally a firm-skill-time effect, in the sample of *stayers*, defined as workers in a given skill group who are employed by the same firm between $t-1$ and t.[24] We define the year-on-year log change in the firm-and-skill-specific wage to be equal to the firm-skill-year fixed effect, that is, the average log change in wages; this approach controls for time-varying returns to observable characteristics and for time-invariant unobservables (including the quality of the worker-firm match). We chain these year-on-year changes together to obtain five-year changes. We measure five-year changes in employment (Δx_{hit} and $\Delta x_{\ell it}$), by deflating changes in firm-and-skill-specific wage bills by the corresponding wage change (Δw_{hit} and $\Delta w_{\ell it}$).

To construct measures of the firm-level equipment stock, we rely on data on firm investments in equipment products, obtained from BRN, and on transaction-level data on the imports of equipment products, obtained from the customs dataset.[25] We first deflate the investment values by a firm-level investment price index. To this end, we first aggregate the transaction-level import data to compute quantities and unit values for the im-

[24] We exclude from the data atypical workers (e.g., cross-border workers, interns and trainees) and those who did not meet a minimum hours-worked threshold.

[25] In the customs data, we define equipment products as those falling into the following categories of the BEC (Rev. 4) classification: *4 – Capital goods (except transport equipment), and parts and accessories thereof*; *51 – Transport equipment, passenger motor cars*; and *521 – Transport equipment, other, industrial*. For more details, see Section C.1.1 in the Appendix.

ports in each equipment product classification and from each origin country for all firms. We then average year-on-year log changes in unit values for each firm across equipment product codes and origins, weighted by Sato-Vartia weights. Since we do not observe the product-level composition of firms' domestic investment, we use the series of production price indices for equipment goods from the French national accounts for all firms' domestic equipment purchases. Then, we chain the firm-level equipment price changes to construct a firm-specific equipment price level in each year (P^e_{it}). We obtain equipment investment quantities by dividing firm-level equipment investment values by the firm-level equipment price. Finally, we construct our measures of equipment stocks by accumulating the investment quantities over time using the perpetual inventory method. We compute five-year log changes in the stocks of equipment from the resulting series of firm-and-year-specific equipment stocks.

We construct a firm-level user cost (rental price) of equipment capital, W_{eit}, as the firm's equipment price index multiplied by the standard Hall-Jorgenson real user cost of capital: the required return plus economic depreciation, minus expected equipment-price appreciation (capital gains), where expected appreciation is proxied by the average change in the sectoral log equipment price over a multi-year window.

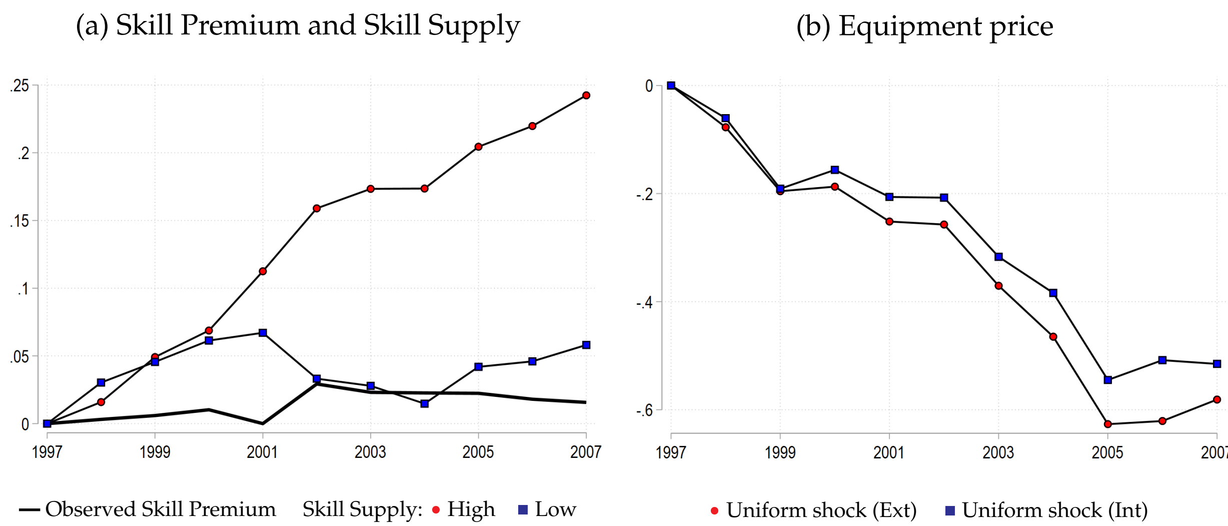
Finally, we obtain payments to capital equipment by multiplying the firm-level user cost of equipment with the measure of equipment stock. Since the coverage of the variables required to construct equipment prices and stocks is lower compared to employment variables, we impute capital equipment expenditures to extend the sample of firms used in the aggregation exercise of Section 5.

3.3 Reduced-Form Facts

In this section, we document aggregate patterns in the evolution of the skill premium and the relative supply of skilled labor in the French economy to argue that the relative demand for skill must have risen substantially over time; and we show that the decline in the price of equipment capital is consistent with the hypothesis that capital-skill complementarity may explain some of this demand shift. We then leverage the decomposition in equation (16) to present moments in the data that shape the strength of capital-skill complementarity.

Aggregate Patterns. Panel (a) of Figure 1 displays the evolution of the observed skill premium and the supply of high- and low-skilled labor in the aggregate French economy between 1997 and 2007. Over this 11-year period, the relative supply of skilled labor rises substantially (similar to the pattern in the U.S., as documented in, e.g., Acemoglu and Autor, 2011) whereas the skill premium is essentially flat. Holding demand fixed, an increase in the relative supply of skilled labor places downward pressure on the skill premium, as shown in equation (A.9). The fact that the skill premium does not fall then implies a substantial increase in the relative demand for skilled labor over the same period.

Figure 1: Aggregate Patterns



Notes: Panel (a) plots the evolution of the observed skill premium and skill supply, with both series constructed using compositionally adjusted aggregate skilled and unskilled wages and the implied compositionally adjusted hours, as described in Appendix C.2.3. Panel (b) plots the cumulated equipment shocks observed over the period, rescaled by the aggregate change in the low-skilled wage proxied with the change in the nominal hourly minimum wage sourced from the French Ministry of Labor. The uniform shock (Ext), represented with red circles, is computed as the log change in the rental price using industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. The uniform shock (Int), represented with blue squares, is computed by aggregating the firm-level heterogeneous changes in the user cost of equipment (W_{eit}). See Appendix C.1.7 for further details.

Panel (b) of Figure 1 shows that the price of equipment (relative to the wage of low-skilled labor, which serves as our numeraire in Section 2) falls substantially over the same time frame. In the presence of aggregate capital-skill complementarity, falling equipment prices increase the relative skill demand and help explain why the skill premium remained flat despite the large increase in skill supply.

Since the skill premium is nearly constant, the only factor-price change leading to a change in aggregate demand is the relative price of equipment. Hence, according to equation (5), we can heroically calibrate the aggregate degree of capital-skill complementarity under the identification assumption that there is no factor-biased technical change that is not embodied in capital equipment (an assumption we do not impose in our micro-to-macro approach). In this case, we can measure the aggregate degree of capital-skill complementarity by dividing $\Delta \log(X_h/X_\ell)$ by $-\theta_e \Delta \log W_e$ using a single long difference between 1997 and 2007 and defining θ_e at its average value over this horizon. This approach yields $\sigma_{\ell e} - \sigma_{he} \approx 0.63$. In Section 5.3 we estimate the aggregate degree of capital-skill complementarity imposing a nested CES aggregate production function and continuing to assume an absence of skill-biased technical change not embodied in equipment; there, we find a similar aggregate estimate. In contrast to this overly simplified approach, the one proposed in our paper allows for skill-biased technical change and leverages very different sources of variation, to which we now turn.

Within Firm Capital-Skill Complementarity. The impact of within-firm capital-skill complementarity on the aggregate, corresponding to the first term in equation (16), is determined by $\sigma_{\ell e,i} - \sigma_{he,i}$. To provide initial evidence for this channel, we estimate the following expression,

$$\Delta x_{\ell it} - \Delta x_{hit} = \alpha_{s_i t} + \beta\, \overline{\theta}_{eit} \Delta w_{eit} + \nu_{it} \tag{23}$$

where $\overline{\theta}_{eit}$ is the average equipment intensity of firm i in years $t-5$ and t. Equation (23) relates the change in firm i's relative employment of low- and high-skilled labor between $t-5$ and t to the interaction between its equipment intensity and the change in its equipment price between $t-5$ and t. Given exclusion restrictions, the parameter β identifies an average—across firms and time—of the strength of firm-level capital-skill complementarity, $\sigma_{\ell e,i} - \sigma_{he,i}$, as shown in the definition of $\sigma_{ff',i}$ in equation (11). In equation (23), $\alpha_{s_i t}$ is a sector-year effect, with s_i denoting firm i's primary sector; hence, we explicitly control for common changes in technology, such as sector-specific (or aggregate) and time-varying skill-biased technical change. We winsorize the dependent variable at the 1st and 99th percentiles within each year and two-digit sector pair.

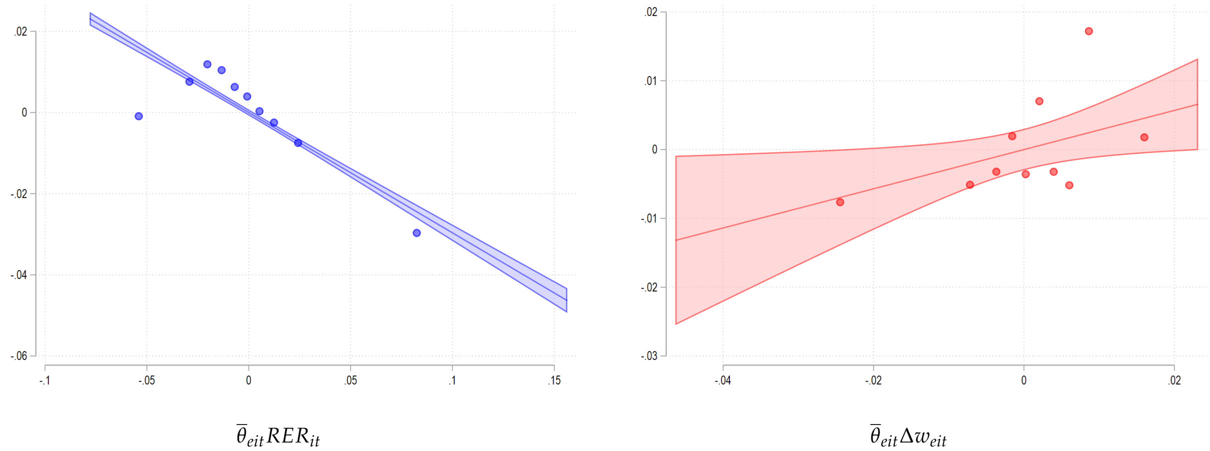
Changes in firm-level equipment prices may be correlated with the residual, since the latter contains firm-specific skill-biased technical change as well as changes in other firm-specific factor prices (both relative to the industry-time average). To address this issue, we instrument for $\overline{\theta}_{eit} \Delta w_{eit}$.[26] Our baseline instrument is $\overline{\theta}_{eit} RER_{it}$. We refer to RER_{it} as the real-exchange-rate instrument. It leverages differences in firm-and-year-specific import exposures to different source countries and movements in real exchange rates between France and these countries and is plausibly uncorrelated with firm-specific factor-biased technical change. We define it formally as

$$RER_{it} = \sum_c s^{pre}_{ic,t} \Delta \log \left(NER_{ct} \cdot \frac{Defl_{FR,t}}{Defl_{ct}} \right), \tag{24}$$

[26] In Section 4 we derive related expressions, in the context of particular forms for firm-level production functions, in which we solve explicitly for the structural residuals.

Figure 2: Within-Firm Capital-Skill Complementarity: 1st and 2nd Stages

(a) 1st Stage (b) 2nd Stage



Notes: The left panel displays a binned scatter plot representation of the residualized first stage (residualizing on two-digit sector-year fixed effects) in which we instrument for $\overline{\theta}_{eit}\Delta w_{eit}$ with $\overline{\theta}_{eit}RER_{it}$. The right panel displays a binned scatter plot representation of the residualized second stage associated with estimating equation (23) using the predicted values of $\overline{\theta}_{eit}\Delta w_{eit}$. The figures display 90% confidence intervals.

where $s^{pre}_{ic,t} = \sum_{\tau=5}^{6} M_{ci,t-\tau} / \sum_{\tau=5}^{6} M_{i,t-\tau}$. Here, $\Delta \log$ indicates five-year log changes, NER_{ct} is the nominal exchange rate (defined as country c's currency per euro), $Defl_{ct}$ is the GDP deflator in the origin country c, and $Defl_{FR,t}$ is the French GDP deflator in year t. The exposure share $s^{pre}_{ic,t}$ is constructed as firm i's import value from country c in years $t-5$ and $t-6$, relative to firm i's total imports over the same two-year window. These pooled two-year pre-period shares reduce sensitivity to idiosyncratic import realizations in any single year and sum to one across origin countries for each importing firm. This avoids what Borusyak et al. (2022) refer to as "incomplete shares" and implies that identification arises not from differences across firms in their import intensity, but instead from differences in the origins from which firms import. A real depreciation of country c's currency relative to the euro is reflected in an increase in RER_{it}, and disproportionately so for firm i if it sources a larger share of its imports from country c in $t-5$ and $t-6$. Intuitively, if firm i obtains a large share of its imports from a given source country c and the real exchange rate of country c depreciates relative to France, then firm i's import cost falls, thereby reducing its cost of equipment.

The instrument is strong: the first-stage KP F statistic is approximately 200. Moreover, it moves $\overline{\theta}_{eit}\Delta w_{eit}$ in the expected direction. The left panel of Figure 2 displays a binned scatter plot representation of the first stage. After residualizing on two-digit sector-year fixed effects, a higher value $\overline{\theta}_{eit}RER_{it}$ is associated with a lower value of $\overline{\theta}_{eit}\Delta w_{eit}$.

Table 1: Within-Firm Capital-Skill Complementarity

	(1)	(2)	(3)	(4)
$\overline{\theta}_{eit}\Delta w_{eit}$	0.28	0.36	0.13	0.19
	(0.17)	(0.16)	(0.17)	(0.16)
Observations	101,087	100,998	68,377	68,318
Year-Sector FE	2 dig	1 dig	2 dig	1 dig
IV	$\overline{\theta}_e RER$	$\overline{\theta}_e RER$	$\overline{\theta}_e RER^{\mathrm{e}}$	$\overline{\theta}_e RER^{\mathrm{e}}$
KP F-Stat	213.99	260.74	205.03	215.12
P-Value	0.10	0.02	0.43	0.24

Notes: This table reports regression coefficients for equation (23) estimated by 2SLS using the RER instrument in columns 1 and 2 and the RER^e instrument in columns 3 and 4. All columns include year-sector effects with sectors defined at the 2-digit level (in columns 1 and 3) or at the 1-digit level (in columns 2 and 4). Robust standard errors are reported in parentheses. KP F-Stat is the Kleibergen and Paap (2006) first-stage F-statistic.

Column 1 of Table 1 displays coefficient estimates and robust standard errors associated with estimating equation (23) using 2SLS and including two-digit sector-year fixed effects. Our estimate of $\widehat{\beta} = 0.28$ implies a moderate degree of capital-skill complementarity at the firm level. Column 2 of Table 1 replicates this exercise, replacing two-digit sector-year fixed effects with one-digit sector-year fixed effects. The estimated strength of firm-level capital-skill complementarity rises slightly, to 0.36, and standard errors become slightly smaller. The right panel of Figure 2 displays a binned scatter plot representation of the data underlying the column 1 estimate. The x-axis displays binned values of the prediction of $\overline{\theta}_{eit}\Delta w_{eit}$ arising from the first-stage regression, residualized with respect to two-digit sector-year effects. The y-axis displays the average value of $\Delta x_{\ell it} - \Delta x_{hit}$, also residualized, within each bin. We observe a clear upward-sloping relationship.

Columns 3 and 4 replicate columns 1 and 2 using an alternative instrument, which we refer to as the equipment-only-real-exchange-rate instrument and denote by RER^e. We define this instrument as in equation (24), but construct the exposure shares using only equipment imports in both the numerator and the denominator (i.e., omitting non-equipment imports). The resulting shares therefore sum to one across origin countries for firms with positive equipment imports in the exposure window. Although this instrument is defined for a substantially smaller number of firm-year pairs than the baseline version, it delivers a similarly strong first stage. We obtain estimates of the strength of firm-level capital-skill complementarity of 0.13 and 0.19 with two-digit and one-digit sector-year fixed effects, respectively. These estimates are slightly smaller and less precise than the corresponding estimates in columns 1 and 2.

In summary, we find reduced-form evidence consistent with the existence of capital-skill complementarity at the firm level. Our estimates range between 0.13 and 0.36. How-

Table 2: Across-Firm Capital-Skill Complementarity and Sectoral Equipment Intensity

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
θ_{est}	0.17	0.14	0.15	0.21	0.23	0.04	0.28
	(0.07)	(0.07)	(0.08)	(0.10)	(0.10)	(0.07)	(0.11)
Observations	363	363	363	363	363	363	363
Sector FE		Man	Man	1 dig	1 dig	2 dig	2 dig
Year FE			Yes		Yes		Yes

Notes: This table reports the results of regressing via OLS the within-two-digit sector covariance across firms between equipment intensity and the relative demand for skill on the sector's equipment intensity at time t, θ_{est}. Column 1 includes no controls. Columns 3, 5, and 7 control for year effects. Column 2 and 3 control for a manufacturing sector effect, columns 4 and 5 control for 1-digit sector effects, and columns 6 and 7 control for 2-digit sector effects. Robust standard errors are reported in parentheses.

ever, the empirical specifications considered so far suffer from Uzawa-McFadden impossibility results described above. To address this important limitation, in Section 4 below, we derive empirical specifications for factor demand that feature heterogeneous elasticities of substitution and are compatible with well-defined underlying production functions. Unlike our reduced-form estimation, our structural model introduces additional terms into the estimating equation (to account for changes in the prices of other factors) and allows the strength of capital-skill complementarity to vary across firms and time periods. In spite of these differences, our structural estimates fall within the range of our reduced-form estimates above.

Across-Firm Capital-Skill Complementarity. The impact of between-firm factor reallocation on aggregate capital-skill complementarity, corresponding to the third and fourth terms in equation (16), is shaped by the covariance of equipment intensity and the relative demand for skill, both across firms within sectors and across sectors. Using two-digit sectors, the expectation across sectors of this within-sector covariance is 0.056; we take this expectation each year and average across years. Hence, a decrease in equipment prices will generate an increase in the aggregate demand for skill by reallocating factors toward more equipment-intensive firms within sectors.

What drives the variations across sectors in the across-firm covariance of equipment and skill intensity? Table 2 displays the result of regressing each *sector-level* cross-firm covariance (measured within each two-digit sector in each year) on the sector's equipment intensity in that year. Across different specifications differing in the choices of fixed effects, we find that this covariance is larger within more equipment-intensive sectors.

The between-sector covariance (also averaged across years), on the other hand, is -0.038. Hence, a decrease in equipment prices will generate a decrease in the aggregate demand for skill as it reallocates factors toward more equipment-intensive sectors.

The importance of these within- and between-sector covariances is determined by the elasticities of demand across firms within and across sectors, ε and η, respectively, since higher elasticities are associated with more reallocation toward equipment-intensive firms within sectors and toward equipment-intensive sectors in response to a decline in equipment prices. We estimate these elasticities in Section 4. Since the elasticity within sectors is greater than across, $\varepsilon > \eta$, and since the expectation of the within-sector covariance (0.056) is greater than the (absolute value of the) covariance across sectors (-0.038), equation (16) implies that between-firm reallocation in response to a decline in equipment prices increases the relative demand for skill.

4 The Estimation of Elasticities

Section 2 characterizes the elasticity of the skill premium with respect to equipment prices. To compute this elasticity, in addition to the firm-level distribution of factor intensities we also need the demand elasticity across sectors η, the demand elasticity across firms within each sector ε, and the complete set of elasticities of firm-level production functions, $\sigma_{ff',i}$. In this section, we estimate these demand and production function elasticities. Whereas our general theoretical results apply for any constant-returns-to-scale firm-level production function, in our baseline analysis we assume that the firm-level production function belongs to the CRESH family presented in Example 3 with three factors (low- and high-skilled labor and equipment capital).

We choose CRESH as our baseline rather than nested CES for two reasons. First, nested CES imposes strong and arbitrary restrictions on the strength of capital-skill complementarity whereas CRESH does not, as shown in Section 2.3. Second, the nested CES specification requires an arbitrary choice of the nesting structure, which affects estimation and can affect counterfactual implications; see, e.g., Baum-Snow et al. (2018). Whereas the CRESH production function has clear advantages relative to nested CES in the presence of three factors, it imposes a strong restriction on cross elasticities, $\sigma_{ff',i}$, in the presence of more than three factors (as would any nested CES): estimates of σ_h and σ_ℓ shape not only the sign of firm-level capital-skill complementarity, but also the sign of skill complementarity for any additional factor f.[27] In addition to our reduced-form evidence presented in Section 3.3, we take two alternative approaches—described in Section 4.1—to provide evidence that our choice of CRESH is not driving our results.

[27]Specifically, the strength of factor-f-skill complementarity for any factor $f \neq h, \ell$ is given by $\sigma_{\ell f,i} - \sigma_{hf,i} = \sigma_f \left(\sigma_\ell - \sigma_h\right)/\overline{\sigma}_i$. Hence, if equipment is more complementary with high-skilled than with low-skilled labor, then so is any other factor f.

4.1 Estimating Equations

Demand Elasticities ε and η. Given our assumptions on the nested CES structure of demand, we can jointly estimate the elasticity of substitution across firms within each sector, ε, and the elasticity of substitution across sectors, η. From equation (1), we can write the change in log firm revenue Δr_{it} as

$$\Delta r_{it} = (1-\varepsilon)\ \Delta p_{it} + (\varepsilon - \eta)\ \Delta p_{st} + \alpha_t + \Delta\phi_{it}, \tag{25}$$

where α_t is a time-t fixed effect, Δp_{it} is the change in firm i's log price, Δp_{st} is the change in the log price index within sector s, and $\Delta\phi_{it}$ is the log change in firm i's demand shifter.[28] In our baseline, we estimate equation (25) using five-year changes at the firm level, where $\Delta x_t \equiv x_t - x_{t-5}$ for any variable x. Since markups are constant under our model, we can measure changes in firm-level prices by aggregating changes in firm-level input costs using their respective cost shares and then subtracting changes in firm-level productivity. We directly measure changes in firm-level unit production costs at fixed productivities and include changes in firm-level productivities in the estimation residual. Sector-level price changes are then constructed by aggregating these firm-level price changes across firms, which we similarly break into two parts: measured sector-level unit production costs at fixed productivities and unmeasured sector-level changes in productivity that we include in the estimation residual. Details are provided in Appendix C.4.

CRESH Production Function Elasticities σ_ℓ, σ_h, and σ_e. The production function specified in equation (20) implies that firm i's factor demand satisfies[29]

$$\Delta x_{eit} - \Delta x_{\ell it} = \beta_{\ell t} + \sigma_\ell\ (\Delta w_{\ell it} - \Delta w_{eit}) + \left(\frac{\sigma_\ell}{\sigma_e} - 1\right)\ \left(\tfrac{\varepsilon}{\varepsilon-1}\Delta r_{it} - \Delta x_{eit}\right) + \nu_{\ell it} \tag{26}$$

$$\Delta x_{eit} - \Delta x_{hit} = \beta_{ht} + \sigma_h\ (\Delta w_{hit} - \Delta w_{eit}) + \left(\frac{\sigma_h}{\sigma_e} - 1\right)\ \left(\tfrac{\varepsilon}{\varepsilon-1}\Delta r_{it} - \Delta x_{eit}\right) + \nu_{hit}, \tag{27}$$

where x_{fit} is the logarithm of employment of factor f in firm i at time t and where we have replaced the change in firm i's log output with the change in log revenue (value added) r_{it} using the structure imposed by the CES demand for firm products. The structural residuals $\nu_{\ell it}$ and ν_{hit}, therefore, depend not only on changes in firm-specific factor-augmenting productivities Δz_{fi}, but also on the change in firm i's demand shifter and the change in the relevant sectoral price index. In practice, we estimate this system including sector-time effects, to absorb sector-time factor-biased technical change.

[28] In an alternative specification we impose sector $\times$ time fixed effects and estimate ε alone.
[29] See the proof of equations (26) and (27) in Appendix A.4.1 (on page A18).

In our baseline, we estimate equations (26) and (27) using five-year changes in factor employment, prices, and revenues. We measure firm i's employment of and payment to each factor as described in Section 3.2, computing five-year changes by chaining the year-on-year log changes for each firm. Our approach ensures that the measurement of firm-level wage changes for each skill group is not affected by changes in the composition of the firm's workforce. As in our reduced-form estimation, we winsorize the dependent variables at the 1st and 99th percentiles within sector and year.

In practice, the parameters σ_ℓ, σ_h, and σ_e can be estimated without including output (or revenue) as an independent variable. We estimate them using equations (26) and (27) for two reasons. First, the inclusion of additional factors of production leaves these estimating equations unchanged. Hence, our approach to identifying the strength of firm-level capital-skill complementarity is robust to a production environment featuring additional factors. Second, unlike alternative nonlinear specifications, equations (26) and (27) have log-linear forms and can be estimated using 2SLS, which is an approach we take in robustness.

Alternative production functions and the estimation of $\sigma_{ff',i}$. We consider two alternatives to assuming that the firm-level production function belongs to the CRESH family.

First, instead of specifying the particular parametric form of the firm-level production function, we directly impose constant cross elasticities of substitution (CXES) for all firms and time periods, $\sigma_{ff',i} \equiv \zeta_{ff'}$, as described in Example 4. This assumption is similar to that imposed in estimating a reduced-form measure of within-firm capital-skill complementarity in Section 3.3. However, unlike in Section 3.3, we provide a system of estimating equations very similar to our baseline CRESH system of equations (26) and (27). In this respect, CXES is a hybrid between our reduced-form evidence in Section 3.3 and our baseline CRESH approach. Second, instead of assuming the CRESH parametric form, we impose the nested CES production function of Example 2. In this case, we provide a system of estimating equations from which we estimate the elasticities ς and ρ. We present estimation details for CXES in Appendix C.5 and for nested CES in Appendix C.6.

4.2 Identification Strategy and Instruments

Here, we describe our identification strategy and describe the instruments used in this strategy. Appendix C.3 contains further details.

Estimation Strategy. We first estimate ε and η using equation (25) via GMM. We then use the implied value ε to estimate σ_ℓ, σ_h, and σ_e using equations (26) and (27) via GMM, since the three structural parameters we aim to identify depend on parameter estimates from both equations. We jointly bootstrap both estimation steps so that our confidence intervals for σ_ℓ, σ_h, and σ_e incorporate the dispersion of our estimated values for ε and η. We take a similar approach to estimation and bootstrapping in the case of constant cross elasticities of substitution. Finally, we estimate ς and ρ in the case of nested CES also via GMM.

Endogeneity and Instruments. In general, changes in firm-level log prices Δp_{it}, on the right-hand side of equation (25), may covary with firm-specific changes in demand shifters $\Delta\phi_{it}$, for example because of shocks to the quality of firm output. This creates a standard endogeneity problem, since the residual on the right-hand side of equation (25) is a function of these demand shifters. Moreover, since we proxy for price changes using the component induced by factor-price changes alone, this creates an additional endogeneity problem, since the residuals are also functions of productivity shifters. Similarly, changes in firm-specific factor prices, firm revenue, and firm equipment stocks, on the right-hand-side of equations (26) and (27), may covary with firm-specific factor-augmenting productivities and with changes in firm i's demand shifter and changes in the relevant sectoral price index. This creates an endogeneity problem, since the residuals on the right-hand side of equations (26) and (27) are functions of these.

To address endogeneity in estimating demand elasticities using equation (25), we leverage the real-exchange-rate instrument (RER_{it}) introduced in equation (24). RER_{it} is correlated with firm i's price change but is plausibly independent of changes in its demand and productivity. We also instrument for sectoral price changes in equation (25) using a sectoral aggregation of our firm-specific real-exchange-rate instrument, which we denote by RER_{st}. The definition of RER_{st} is identical to that of RER_{it}, except that firm-time-specific import shares, $s^{pre}_{ic,t}$, are replaced by sector-time-specific import shares, $s^{pre}_{sc,t}$, constructed over the same pre-period window.

To address endogeneity in estimating production function parameters using equations (26) and (27), we leverage the real-exchange-rate instrument as well as three additional instruments. Two of these instruments are intended to shift the wages of skilled and unskilled workers facing each firm; we refer to these as skill-specific-wage instruments and denote them by SSW^h_{it} for skilled labor and SSW^ℓ_{it} for unskilled labor. These instruments leverage differences in the spatial compositions of firm production, the industrial mix of different French regions, and variations in skill intensities and nationwide growth across industries; they are defined formally in Appendix C.3. The final instrumental approach is intended to shift firm revenue and is based on our real-exchange-rate instrument and to the world-export-supply instrument introduced by Hummels et al. (2014); we also refer to it as the world-export-supply instrument and denote it by WES_{it}. This instrument leverages differences in firm-specific intermediate import exposure to different products and source countries, along with the movements in the supplies of those products from the corresponding source countries to the rest of the world; it is defined formally in Appendix C.3.

4.3 Estimation Results

Demand Elasticity Estimates. Table 3 displays estimates of the within-sector, between-firm elasticity of substitution, ε, and the between-sector elasticity of substitution, η, obtained by estimating equation (25) via GMM. Column 1 shows the estimates obtained using each covariate as its own instrument whereas columns 2-5 display the estimates obtained using the instruments indicated in the bottom panel of the table. In our baseline, displayed in column 2, industries are defined at the four-digit level and we use the two RER instruments, at the firm and sector level. Since the instruments require import values for each firm, in our baseline we restrict the sample of firms to those with import values that are at least 1% of their gross output, when averaged over the sample period.

Whereas estimates in column 1 of both ε and η are close to one, our baseline instrumented estimates in column 2 are 4.49 and 3.12. This downward bias is consistent with a positive correlation between demand shocks and prices. The between-firm estimate of 4.49 obtained within four-digit sectors is in the range of values reported by Broda and Weinstein (2006) for products at the five-digit level. Moreover, our instruments are strong. In our baseline specification, the minimum of the two SW first-stage F statistics obtained by running the corresponding 2SLS estimation of equation (25) is above 45; the first stage remains sufficiently strong even in more demanding specifications.

In column 3, we demean all variables by two-digit sector-year means and drop the estimation of η. Here, we obtain a slightly lower value of $\varepsilon = 3.13$. One potential concern with the moment condition in our baseline specification is that firms may import from and export to the same market. In this case, the RER instruments may be correlated with export demand shocks. To address this concern, in column 4 we exclude firms that import from any country to which they also export more than 10% of their total export value over the period. This halves the size of the sample, but the estimates of ε and η remain relatively stable at levels not significantly different from our baseline column 2 estimates. Finally, in column 5 we additionally incorporate the two skill-specific-wage instruments. Our estimates are largely robust, although the first stage in the corresponding 2SLS specification is slightly weaker and the resulting estimates are slightly lower.[30]

[30]Table D.1 in the Appendix replicates Table 3 varying the level of sectoral aggregation. Table D.2 esti-

Table 3: Between-Firm Demand Elasticity Estimates

	(1)	(2)	(3)	(4)	(5)
ε	0.75	**4.49**	3.13	5.44	3.37
	(0.01)	**(0.77)**	(0.36)	(1.32)	(0.50)
η	0.80	**3.12**		3.56	2.47
	(0.02)	**(0.43)**		(0.73)	(0.28)
Observations	162,320	**162,320**	162,284	78,918	162,149
Eta level	4 dig	**4 dig**	-	4 dig	4 dig
Year FE	Yes	**Yes**		Yes	Yes
Year-Sect FE			2 dig		
Excl Exp Mkt	No	**No**	No	Yes	No
IV	-	***RER***	*RER*	*RER*	*RER+SSW*
SW Min F-Stat		**45.80**	139.80	20.44	25.29

Notes: This table reports the results of estimating equation (25) by GMM. The dependent variable is the 5-year change in firm i's value added and the independent variables are corresponding changes in total input costs at the firm and 4-digit sector level. In column 2 and all columns thereafter, we use the firm-level RER instrument defined in equation (24) and the sectoral RER instrument defined in equation (C.1) in Appendix C.3. Additionally, column 5 uses the skill-specific-wage instruments defined in equation (C.2) in Appendix C.3. All variables are expressed as deviations from year means, except in column 3, where they are expressed as deviations from year-by-two-digit-sector means. In column 4, we exclude firms that import from any country in which they also export more than 10% of their total export value over the period. The final row reports the minimum value of the Sanderson and Windmeijer (2016) first-stage F statistics obtained by running the equivalent 2SLS estimation. Robust standard errors are reported in parentheses.

Firm-Level Production Elasticity Estimates (CRESH). Table 4 displays the estimates of the structural parameters of interest in the top panel—σ_ℓ, σ_h, and σ_e—and the corresponding averages (taken within the estimation sample) across firms and time periods of the cross elasticities of interest in the bottom panel—$\overline{\sigma}_{\ell e,i}$, $\overline{\sigma}_{he,i}$, and $\overline{\sigma}_{\ell h,i}$—estimated using GMM while varying the value of ε, the instruments, and the fixed effects. In all specifications we instrument using the two skill-specific wage instruments and the world-export-supply instrument. In columns 1-3 we use the real-exchange-rate instrument whereas in columns 4-6 we use the equipment-only-real-exchange-rate instrument.

Column 1 displays our baseline estimation results, in which we instrument using the RER instrument, we demean all variables from sector-year averages using two-digit sectors, and we use our baseline value of $\varepsilon = 4.49$. We obtain estimates of $\sigma_\ell = 1.06$, $\sigma_h = 0.87$, and $\sigma_e = 1.31$. We find that $\sigma_\ell > \sigma_h$, which implies that equipment is more substitutable with low- than high-skilled labor: at the firm level, production exhibits equipment-skill complementarity. This difference in parameters is statistically significant based on our bootstrapped samples, as shown in the bottom panel of Table 4. These estimates and firm factor intensities imply the average, across firms and time periods, of firm-level capital-skill complementarity, $\overline{\sigma}_{\ell e,i} - \overline{\sigma}_{he,i}$, is approximately 0.23, in the middle of the range of values estimated in our reduced-form exercises and reported in Table 1.

mates ε alone by 2SLS, and controls for η using sector-year fixed effects.

Table 4: Firm-Level Production Elasticities (CRESH)

	(1)	(2)	(3)	(4)	(5)	(6)
σ_ℓ	**1.06**	0.88	1.02	0.92	0.84	0.90
	(0.16)	(0.11)	(0.16)	(0.15)	(0.09)	(0.15)
σ_h	**0.87**	0.79	0.89	0.79	0.75	0.81
	(0.12)	(0.09)	(0.13)	(0.14)	(0.09)	(0.14)
σ_e	**1.31**	1.08	1.23	0.92	0.87	0.91
	(0.27)	(0.16)	(0.23)	(0.20)	(0.11)	(0.18)
$\overline{\sigma}_{\ell e,i}$	**1.29**	1.03	1.20	0.96	0.89	0.94
	(0.24)	(0.14)	(0.21)	(0.18)	(0.10)	(0.17)
$\overline{\sigma}_{he,i}$	**1.06**	0.93	1.04	0.83	0.79	0.84
	(0.18)	(0.12)	(0.17)	(0.16)	(0.10)	(0.16)
$\overline{\sigma}_{\ell h,i}$	**0.85**	0.75	0.87	0.83	0.76	0.83
	(0.11)	(0.08)	(0.12)	(0.14)	(0.09)	(0.14)
Observations	**101,087**	100,998	101,087	68,377	68,318	68,377
Year-Sector FE	**2 dig**	1 dig	2 dig	2 dig	1 dig	2 dig
IV	***RER***	*RER*	*RER*	RER^e	RER^e	RER^e
$\Pr[\sigma_\ell < \sigma_h]$	**0.02**	0.10	0.04	0.07	0.06	0.13
ε	**4.49**	4.49	3.13	4.49	4.49	3.13

Notes: This table reports regression coefficients for the system of equations (26) and (27) estimated using GMM for a given value of ε and with variables demeaned by year-sector means, with sectors defined at the 2-digit or at the 1-digit level in columns 2 and 5. Each reported value of $\overline{\sigma}_{ff',i}$ is constructed by averaging across the firm-year observations used in the estimation sample for that column. All columns use the skill-specific-wage instruments defined in equation (C.2) and the world-export-supply instrument defined in equation (C.3) in Appendix C.3. Additionally, columns 1-3 use the firm-level RER instrument defined in equation (24), while columns 4-6 use the equipment-only-real-exchange-rate instrument, both defined in Section 3.3. Bootstrap standard errors are reported in parentheses, obtained by bootstrapping the estimation of ε and of this system 200 times. $\Pr[\sigma_\ell < \sigma_h]$ reports the share of bootstrapped estimates in which σ_ℓ is lower than σ_h.

The strength of within-firm capital-skill complementarity is robust to sectoral aggregation in the sector-year demeaning, to the value of ε, and to the specific real-exchange-rate instrument employed. Column 2 replicates column 1, but demeans all variables at the sector-year level using 1-digit (instead of 2-digit) sectors, whereas column 3 replicates column 1 but using our lowest estimate of $\varepsilon = 3.13$ from Table 3 (instead of our baseline estimate of $\varepsilon = 4.49$). Columns 4-6 then replicate columns 1-3, but using RER^e as the instrument (instead of RER). Across all columns, we find that within-firm capital-skill complementarity is statistically significant and that the implied value of $\overline{\sigma}_{\ell e,i} - \overline{\sigma}_{he,i}$ ranges from approximately 0.10 to 0.23. In Table D.5 we provide additional robustness varying the set of instruments, the value of ε, and how we deal with outliers.

Alternative Firm-Level Production Elasticity Estimates. Table 5 displays the firm-level cross elasticities of interest that result from our two alternative approaches to estimating $\sigma_{ff',i}$: the common CXES parameters $\zeta_{\ell e}$, ζ_{he}, and $\zeta_{\ell h}$ in columns 1 and 2, and the averages across firms and time periods $\overline{\sigma}_{\ell e,i}$, $\overline{\sigma}_{he,i}$, and $\overline{\sigma}_{\ell h,i}$ implied by the nested CES specification in columns 3 and 4. These results can be compared directly to our baseline CRESH results

Table 5: Firm-Level Production Elasticities, CXES and Nested-CES

	CXES		Nested CES	
	(1)	(2)	(3)	(4)
$\zeta_{\ell e} = \overline{\sigma}_{\ell e,i}$	1.24	0.96	1.21	1.37
	(0.24)	(0.17)	(0.32)	(0.46)
$\zeta_{he} = \overline{\sigma}_{he,i}$	1.06	0.86	1.00	1.06
	(0.17)	(0.15)	(0.05)	(0.18)
$\zeta_{\ell h} = \overline{\sigma}_{\ell h,i}$	0.86	0.87	1.21	1.37
	(0.13)	(0.16)	(0.32)	(0.46)
Observations	101,087	68,377	400,310	100,961
IV	*RER*	*RER*^{e}	*SSW*	*SSW*
Sample	CRESH	CRESH	Full	CRESH
$\Pr[\zeta_{\ell e} < \zeta_{he}] = \Pr[\overline{\sigma}_{\ell e,i} < \overline{\sigma}_{he,i}]$	0.04	0.17	0.26	0.22

Notes: The first two columns of the table (CXES) report the estimated common CXES parameters $\zeta_{\ell e}$, ζ_{he}, and $\zeta_{\ell h}$ from the system of equations (C.6) and (C.7) in Appendix C.5, estimated using GMM for the baseline value of $\varepsilon = 4.49$ and with variables demeaned by year-sector means, with sectors defined at the 2-digit level. The last two columns (Nested-CES) report average values of $\sigma_{ff',i}$ across the firm-year observations in each column's estimation sample, obtained from GMM estimates of the underlying parameters ς and ρ from equations (C.8) and (C.9) in Appendix C.6. Variables are demeaned by year-sector means, with sectors defined at the 4-digit level. Column 4 reports the estimates on the smaller sample of firms used for the CRESH and CXES estimation, with a small difference in the number of observations due to dropped singletons. All columns use the skill-specific-wage instruments defined in equation (C.2). Columns 1 and 2 use two additional instruments: (i) the world-export-supply instrument defined in equation (C.3) in Appendix C.3 and (ii) the firm-level RER instrument defined in equation (24) for column 1 or the equipment-only-real-exchange-rate instrument defined in Section 3.3 for column 2. Bootstrap standard errors are reported in parentheses, obtained by bootstrapping the estimation of ε and/or the respective systems 200 times. The row labeled "Pr" reports the share of bootstrapped estimates in which $\zeta_{\ell e} < \zeta_{he}$ for the CXES columns and $\overline{\sigma}_{\ell e,i} < \overline{\sigma}_{he,i}$ for the Nested-CES columns.

in the bottom panel of Table 4.

Columns 1 and 2 present results in which we impose constant cross elasticities of substitution (CXES) for all firms and time periods, $\sigma_{ff',i} \equiv \zeta_{ff'}$. Equations (C.6) and (C.7) in Appendix C.5 display the system of equations from which we estimate the common firm-level CXES parameters $\zeta_{\ell e}$, ζ_{he}, and $\zeta_{\ell h}$. In column 1 we display results in which we instrument using the two skill-specific wage instruments, the world-export-supply instrument, and the real-exchange-rate instrument. In column 2 we replace the real-exchange-rate instrument with its equipment-only counterpart; Table D.6 in Appendix D displays results of additional robustness exercises.

In columns 3 and 4, we impose the nested CES production function of equation (17). Equations (C.8) and (C.9) in Appendix C.6 display the system of equations from which we estimate the production function parameters ς and ρ. In estimating this system, we use only the skill-specific wage instruments. Hence, we do not need to restrict the sample to importing firms, which yields a much larger sample than in our baseline CRESH estimation in Table 4 or in the CXES estimation of columns 1 and 2 in Table 5. In column 3 we display results obtained when using this larger sample. Column 4 replicates column 3, but on the smaller sample of firms used for the CRESH and CXES estimation.

Across all columns, we obtain point estimates that are consistent with within-firm capital-skill complementarity: $\zeta_{\ell e} > \zeta_{he}$ in the CXES columns and $\overline{\sigma}_{\ell e,i} > \overline{\sigma}_{he,i}$ in the Nested-CES columns. These estimates imply within-firm capital-skill complementarity ranges from 0.10 to 0.31, similar to our reduced-form evidence in Section 3.3 and our CRESH estimates in Table 4. Finally, the estimates are statistically significant in column 1, although not in the other columns.

5 Quantification

We now use our framework to study the implications of observed shocks to equipment prices (and factor supplies) on the skill premium. In our baseline, we consider a setting featuring CRESH firm-level production functions in which the economy faces a uniform fall in the relative price of equipment. We compute the response of the skill premium using the results developed in Section 2.2 and the full "aggregation sample" of firms. In robustness exercises we consider both constant cross elasticities of substitution and nested-CES firm-level production functions. We then use the extensions developed in Section 2.4 to study the impacts of heterogeneity in price shocks observed in our firm-level data and of an alternative model of nested consumer demand featuring three (rather than two) levels of aggregation.

5.1 Baseline Results

We first compute the aggregate degree of capital-skill complementarity and the elasticity of the skill premium with respect to the equipment price, each calculated as averages across sample years. We then solve for the implications of the observed decline in the equipment price.

Capital-Skill Complementarity. Table 6 displays the aggregate degree of capital-skill complementarity $\sigma_{\ell e} - \sigma_{he}$ defined in equation (16) and the elasticity of the skill premium with respect to equipment prices $d\log(W_h/W_\ell)/d\log W_e$ constructed using equation (6), each calculated using shares in each year and then averaging across all years.

Column 1 is our baseline specification, corresponding to the micro elasticities presented in column 1 of Table 4. The results imply that our measure of capital-skill complementarity is approximately 0.24 and that a decrease in the equipment price of 1% increases the skill premium by approximately 0.07%. Column 2 uses values of σ_ℓ, σ_h, and σ_e from column 2 of Table 4, which are estimated within 1-digit sector and year. Column 3 replaces ε with the lowest estimate from Table 3 and uses the corresponding estimates of

Table 6: Baseline Aggregate Elasticities

	(1)	(2)	(3)	(4)	(5)	(6)
$\sigma_{\ell e} - \sigma_{he}$	0.24	0.18	0.22	0.19	0.17	0.19
	[0.14, 0.35]	[0.11, 0.24]	[0.14, 0.31]	[0.11, 0.29]	[0.12, 0.23]	[0.12, 0.28]
$\frac{d\log(W_h/W_\ell)}{d\log W_e}$	-0.07	-0.06	-0.09	-0.06	-0.06	-0.09
	[-0.11, -0.04]	[-0.08, -0.04]	[-0.13, -0.06]	[-0.09, -0.03]	[-0.08, -0.04]	[-0.13, -0.05]
ε	4.49	4.49	3.13	4.49	4.49	3.13
η	3.12	3.12	0.50	3.12	3.12	0.50
σ_ℓ	1.06	0.88	1.02	0.92	0.84	0.90
σ_h	0.87	0.79	0.89	0.79	0.75	0.81
σ_e	1.31	1.08	1.23	0.92	0.87	0.91

Notes: $\sigma_{\ell e} - \sigma_{he}$ is the aggregate degree of capital-skill complementarity and $d\log(W_h/W_\ell)/d\log W_e$ is the elasticity of the skill premium with respect to the equipment price. These elasticities are constructed assuming the firm-level production function belongs to the CRESH family and using firm factor shares in each year and then averaging across all sample years. Column 1 is our baseline, columns 2-6 use our estimates of σ_ℓ, σ_h, and σ_e from the corresponding columns of Table 4. The table displays 90% confidence intervals obtained by bootstrapping the estimation of each σ_f 200 times.

σ_ℓ, σ_h, and σ_e from column 3 of Table 4. We do not estimate the elasticity of substitution across sectors in that specification, so we set $\eta = 0.5$, as in Buera and Kaboski (2009). Finally, columns 4–6 use the parameter estimates from the corresponding columns of Table 4, in which we use RER^e as the instrument. The qualitative insight that a decline in the price of equipment raises the skill premium is robust across specifications. Quantitative elasticities are largely robust as well, with the elasticity of the skill premium ranging from -0.06 to -0.09 and the aggregate degree of capital-skill complementarity from 0.17 to 0.24.

Table 7 decomposes these elasticities into within-firm and across-firm components using equation (16), focusing on our baseline specification of Column 1 in Table 6.[31] We find that within-firm capital-skill complementarity accounts for a large share of the aggregate elasticities, with within-firm equipment-labor substitution and cross-firm reallocation roughly offsetting each other.

Capital-Skill Complementarity and the Evolution of the Skill Premium. Next, we use our framework to quantify the contribution of the observed fall in the relative price of equipment in France to the evolution of the skill premium in our sample period. In addition to computing the elasticity of the skill premium with respect to the equipment price separately in each year of the data, we also need to measure the change in the equipment price between each year. Since we take the wage of low-skilled labor as the numeraire

[31] The decomposition for the skill premium is obtained by combining the decomposition of the degree of capital-skill complementarity in equation (16) with the solution for the skill premium in equation (6).

Table 7: Decomposition of Baseline Aggregate Elasticities

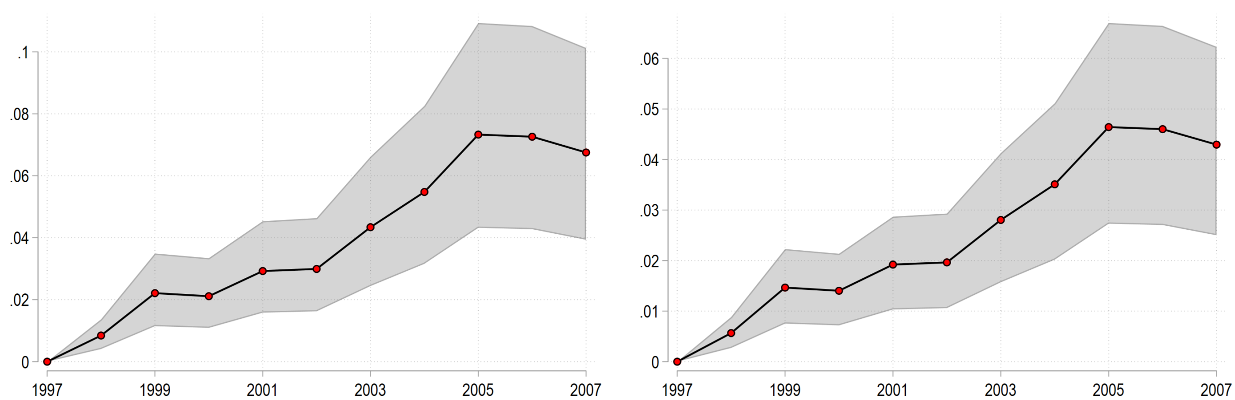
	Capital-Skill Compl	Skill Premium
	$\sigma_{\ell e} - \sigma_{he}$	$\frac{d \log(W_h/W_\ell)}{d \log W_e}$
Aggregate Elasticity	0.24	-0.07
	[0.14, 0.35]	[-0.11, -0.04]
Within firm complementarity	0.21	-0.07
	[0.04, 0.41]	[-0.12, -0.01]
Within firm substitution	-0.11	0.03
	[-0.20, -0.04]	[0.01, 0.06]
Cross firm	0.14	-0.04
	[0.14, 0.14]	[-0.05, -0.04]

Notes: $\sigma_{\ell e} - \sigma_{he}$ and $d \log(W_h/W_\ell)/d \log W_e$ are the elasticities of aggregate capital-skill complementarity and the skill premium with respect to the equipment price. The table decomposes the baseline estimates of Table 6 into within-firm and cross-firm components according to equation (16). These elasticities are constructed using firm factor shares in each year and then averaging across all sample years. The decomposition of the skill-premium elasticity is not exact due to rounding error. The table displays 90% confidence intervals obtained by bootstrapping the estimation of each σ_f 200 times.

Figure 3: Predicted Skill Demand and Skill Premium, Uniform Shock

(a) Predicted Skill Demand

(b) Predicted Skill Premium



Notes: The figure plots the evolution of the predicted skill demand (panel a) and skill premium (panel b) in response to the observed fall in the equipment price relative to the low-skilled shadow wage, using the baseline estimates from column 1 of Table 6. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low-skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labor. See Appendix C.1.7 for further details. The figure displays a 90% confidence interval obtained by bootstrapping the estimation of each σ_f 200 times.

in our theory in Section 2, we measure the change in equipment prices relative to the low-skilled wage.

We measure year-on-year changes in the equipment price by aggregating industry-specific equipment investment prices taken from French national accounts and using industry-specific depreciation rates from EU-KLEMS. We measure the low-skilled wage as the nominal hourly minimum wage, sourced from the French Ministry of Labor (see Appendix C.1.7 for further details).[32] Based on this data, the cumulative decline in the relative equipment price is approximately 58 log points over the period 1997-2007, with the fall in equipment prices and the rise in low-skilled wages each accounting for roughly half of this decline; see details in Table D.8 in the appendix.

Panel (a) of Figure 3 shows the model-predicted impact of the fall in the relative equipment price on skill demand between 1997-2007, computed by accumulating the predicted year-on-year response according to equation (5), along with the associated 90% confidence interval. Panel (b) shows the model-predicted impact of the fall in the relative equipment price on the skill premium over the same time period. To compute the predicted change in the skill premium between years $t-1$ and t, we compute aggregate elasticities using the distribution of factor intensities in year $t-1$ and t, and use the relative equipment price change between $t-1$ and t as the corresponding shock.[33] We then compute the implied cumulative change in the skill premium starting from 1997. The confidence interval is constructed by re-estimating the micro-level elasticities of substitution across bootstrapped samples of the data (200 times). Between 1997 and 2007, the decline in the price of equipment relative to the wage of low-skilled labor generates a 4 log point increase in the skill premium, which is statistically different from zero.

We obtain similar results when we perform a simpler exercise multiplying the total cumulative change in the relative equipment (rental) price by the average aggregate elasticity over the entire period from Column 1 of Table 6. The results are displayed in Table D.8 and yield similar conclusions. At the same time, this approach allows us to extend the time frame of the analysis to periods in which our micro data are not available, to obtain the predicted changes over almost three decades. According to our model, the decline in equipment prices generates an approximately 8% increase in the skill premium in France between 1990 and 2015.

Table 8 decomposes the predicted change in the skill premium, confirming the results previously shown for the decomposition of the aggregate elasticities. The within-firm complementarity component accounts for essentially all of the predicted change, with within-firm equipment-labor substitution and across-firm reallocation roughly offsetting each other.

Panel (a) of Figure 4 displays our model's predicted change in the skill premium taking into account not only changes in equipment prices—as in panel (a) of Figure 3—but

[32]We obtain very similar results by using the changes in the hourly wage of low-skilled workers from the Socio-Economic Accounts of the WIOD database (2013 Release).

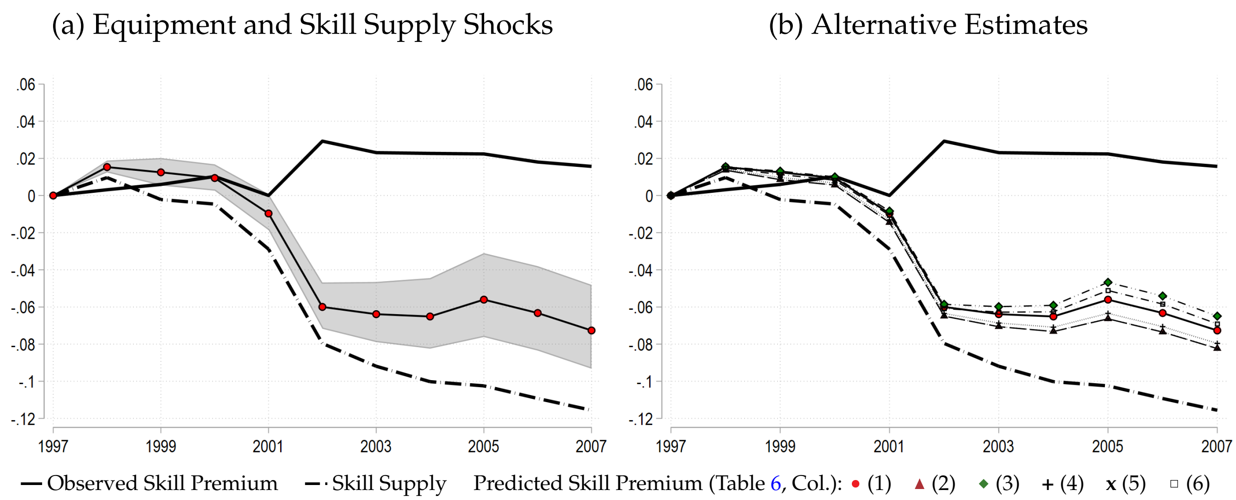
[33]To compute the cumulative predicted change, we rely on second-order approximation results presented in Appendix A.5.

Table 8: Decomposition of the Predicted Change in the Skill Premium, 1997-2007

Total	W/in firm compl.	W/in firm subst.	Cross firm
0.04	0.04	-0.02	0.02
[0.03, 0.06]	[0.01, 0.07]	[-0.03, -0.01]	[0.02, 0.03]

Notes: This table reports the decomposition of the predicted log change in the skill premium in response to the observed fall in the equipment price relative to the low-skilled shadow wage. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low-skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labor. See Appendix C.1.7 for further details. The table displays 90% confidence intervals obtained by bootstrapping the estimation of each σ_f 200 times.

Figure 4: Predicted Skill Premium vs Observed Skill Premium and Skill Supply



Notes: Panel (a) plots the evolution of the predicted skill premium ($d\log(W_h/W_\ell)$) in response to the observed fall in the relative equipment price and the observed change in the skill supply. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low-skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labor. See Appendix C.1.7 for further details. The figure displays a 90% confidence interval obtained by bootstrapping the estimation of each σ_f 200 times. Panel (b) plots the variation in the predicted skill premium across the different estimated aggregate elasticities of Table 6.

also changes in the relative supply of skill using equation (A.9) from Appendix A.3.1. Figure 4 additionally displays the change in the skill premium observed in our data. The substantial rise in the relative supply of skilled labor leads to a predicted decline in the skill premium. The size of this decline is close to the size of the change in the relative skill supply because $\sigma_{\ell h}$ is close to 1. The decline in equipment prices and capital-skill complementarity push in the opposite direction, but are not sufficient to counteract the impact of the rise in the supply of skill on the skill premium: together, changes in skill supply and in equipment prices are predicted to lead to a fall in the skill premium, whereas the observed skill premium is broadly stable over time. The remaining gap between the model's predicted change in the skill premium in response to changing equipment prices and skill supply and the observed evolution of the skill premium can be attributed to all remaining shocks, including skill-biased technical change that is not embodied in capital equipment and changes in wedges between different inputs. Panel (b) displays the robustness of our quantitative results across the estimation specifications displayed in Table 4. Consistent with the elasticities in Table 6, we obtain very similar results across specifications.

5.2 Alternatives and Extensions

5.2.1 Constant Cross Elasticities of Substitution (CXES)

Columns 1 and 2 of Table 9 replicate Table 6, using the estimated CXES parameters from columns 1 and 2 of Table 5. These estimates result in aggregate degrees of capital-skill complementarity of 0.22 and 0.18 and elasticities of the skill premium with respect to equipment prices of -0.07 and -0.06. These aggregate elasticities are very similar to the CRESH case.

As described in Example 4, the CXES assumption has a particularly clean aggregation property, which helps clarify how elasticity estimates shape the degree of aggregate capital-skill complementarity. In the case of CXES, equation (16) simplifies dramatically, with the aggregate strength of capital-skill complementarity depending on four moments in the data and four elasticities. Taking an average across time of each of these moments in our data yields

$$\sigma_{\ell e} - \sigma_{he} = 0.62\ (\zeta_{\ell e} - \zeta_{he}) - 0.02\ \left(\tfrac{1}{2}\,\zeta_{\ell e} + \tfrac{1}{2}\zeta_{he}\right) + 0.06\,\varepsilon - 0.04\,\eta \tag{28}$$

We can conduct comparative statics on the aggregate degree of capital-skill complementarity analytically using the previous expression. Consider varying the strength of within-firm capital-skill complementarity. To do so, use the baseline values of $\varepsilon = 4.49$ and $\eta = 3.12$, so that the two across-firm reallocation channels sum to 0.14. And set the average of $\zeta_{\ell e}$ and ζ_{he} to one, which yields a within-firm equipment-labor substitution channel equal to -0.02, as is approximately true in columns 1 and 2 of Table 9. Now consider the impact on the degree of aggregate capital-skill complementarity of varying the common degree of firm-level capital-skill complementarity from 0.10 (which is the smallest of our estimates, obtained in column 5 of Table 4), to 0.18 (as in column 1 of Table 9), to 0.36 (which is the largest of our estimates, obtained in column 2 of Table 1). Then $\sigma_{\ell e} - \sigma_{he}$ varies from 0.18, to 0.23 (which slightly differs from its value in column 1 of Table 9 due to rounding error), to 0.34. Of course, the previous expression can be combined with

Table 9: Aggregate Elasticities – CXES and Nested-CES firm production functions

	CXES		Nested CES	
	(1)	(2)	(3)	(4)
$\sigma_{\ell e} - \sigma_{he}$	0.22	0.18	0.21	0.27
	[0.12, 0.35]	[0.10, 0.28]	[-0.08, 0.37]	[-0.14, 0.45]
$\frac{d\log(W_h/W_\ell)}{d\log W_e}$	-0.07	-0.06	-0.06	-0.07
	[-0.11, -0.04]	[-0.09, -0.03]	[-0.09, 0.03]	[-0.10, 0.05]
ε	4.49	4.49	4.49	4.49
η	3.12	3.12	3.12	3.12
$\zeta_{\ell e} = \overline{\sigma}_{\ell e,i}$	1.24	0.96	1.21	1.37
$\zeta_{he} = \overline{\sigma}_{he,i}$	1.06	0.86	1.00	1.01
$\zeta_{\ell h} = \overline{\sigma}_{\ell h,i}$	0.86	0.87	1.21	1.37

Notes: $\sigma_{\ell e} - \sigma_{he}$ is the aggregate degree of capital-skill complementarity and $d\log(W_h/W_\ell)/d\log W_e$ is the elasticity of the skill premium with respect to the equipment price. These elasticities are constructed using firm factor shares averaged across all sample years. The columns replicate the exercises in Table 5. The table displays 90% confidence intervals obtained by bootstrapping the estimation of the corresponding CXES or nested-CES production-function parameters 200 times.

any other values of this firm-level elasticity or any of the other elasticities to solve for alternative values of the aggregate elasticity.

5.2.2 Nested CES (KORV) Specification

We repeat the same exercise with the nested CES production function. Columns 3 and 4 of Table 9 replicate Table 6, using the estimates of the nested CES production function from columns 3 and 4 of Table 5. The estimates of the micro-elasticities ς and ρ used in columns 3 and 4 result in aggregate degrees of capital-skill complementarity of 0.21 and 0.27 and elasticities of the skill premium with respect to equipment prices of -0.06 and -0.07. These aggregate elasticities are very similar to the CRESH case.

Given the similarity of results in CRESH, CXES, and nested CES, we conclude that our findings are not driven by specific functional form assumptions. Moreover, because the aggregate degree of capital-skill complementarity is similar in CRESH and nested CES, yet the estimation sample is much larger in the case of nested CES (including firms that are not heavily active in import markets), we additionally conclude that our baseline results are not driven by the restricted estimation sample.

Table 10: Decomposition of CXES Aggregate Elasticities

	Capital-Skill Compl	Skill Premium
	$\sigma_{\ell e} - \sigma_{he}$	$\frac{d\log(W_h/W_\ell)}{d\log W_e}$
Aggregate Elasticity	0.22	-0.07
	[0.12, 0.35]	[-0.11, -0.04]
Within firm complementarity	0.11	-0.04
	[0.00, 0.24]	[-0.07, -0.00]
Within firm substitution	-0.02	0.01
	[-0.03, -0.02]	[0.01, 0.01]
Cross firm	0.14	-0.04
	[0.14, 0.14]	[-0.05, -0.04]

Notes: $\sigma_{\ell e} - \sigma_{he}$ and $d\log(W_h/W_\ell)/d\log W_e$ are the elasticities of aggregate capital-skill complementarity and the skill premium with respect to the equipment price. The table decomposes the baseline CXES estimates of Table 9 (col. 1) into within-firm and cross-firm components according to equation (16). These elasticities are constructed using firm factor shares in each year and then averaging across all sample years. The decompositions are not exact due to rounding error. The table displays 90% confidence intervals obtained by bootstrapping the estimation of each $\zeta_{ff'}$ 200 times.

5.2.3 Heterogeneous Shocks to Equipment Prices

We next use the theory presented in Section 2.4 to account for the heterogeneity in shocks experienced by different firms. In particular, we measure firm-specific equipment prices as outlined in Appendix A.3.2. We then construct model-consistent year-on-year changes in the skill premium following the methodology presented in Section 2.4.2.

The results of the exercise are shown in Figure D.1 and in Table D.9 in the Online Appendix. Compared to the case of the uniform shock studied before, the decline in the firm-specific relative price of equipment now generates a larger 5 log point increase in the skill premium. While the within-firm complementarity component accounts for a large share of the predicted increase in the skill premium as in the case of uniform shocks, the increase in the prediction is mostly driven by the cross-firm component. This result is driven by the positive covariance between price changes and skill intensity, which is particularly strong after 2002 when skill and equipment intensive firms experienced large declines in the relative price of imported equipment (see Figures D.2 and D.3).[34]

[34]For a better comparison with the heterogeneous price shocks, Table D.9 also displays the same decomposition using a uniform shock obtained by aggregating the firm-level heterogeneous shocks to the user cost of equipment, but the conclusions from the decomposition are virtually identical when using the equipment price changes from French national accounts as in Table 8. See Figure 1b for a comparison between the two aggregate uniform shocks and Section C.1.7 for more details on their construction.

5.2.4 Aggregation with Three-Level Nested CES Preferences

We perform a final additional exercise in which the CES preferences are defined over three nests, with a top nest defined at the 1-digit sector level. We set the elasticity of substitution across 1-digit sectors to 0.5, as in Buera and Kaboski (2009). Figure D.4 shows that results are robust both for our baseline CRESH specification as well as the alternative estimates in the remaining columns of Table 6. In the exercise we have kept the same estimates for the firm-level elasticities, and in the Online Appendix we show that the estimates remain very similar when the demand elasticities across firms and sectors (ε and η) are re-estimated controlling for 1-digit-sector-by-year fixed effects (see Tables D.3 and D.7).

5.3 Comparison with the Prior Literature

In our exercises above, we obtain measures for the aggregate degree of capital-skill complementarity, defined as $\sigma_{\ell e} - \sigma_{he}$, in the range of 0.17 to 0.27, and an elasticity of the skill premium with respect to the equipment price in the range of -0.06 to -0.09; see Tables 6 and 9. How do these results compare to prior work in the literature?

In a seminal contribution to the literature, Krusell et al. (2000) (KORV) used aggregate data on skill supply, the skill premium, and aggregate factor intensities from the U.S. to estimate a nested CES aggregate production function in line with Example 2. The core identification assumption behind their approach rules out the presence of trends in skill-augmenting productivity that are not embodied in capital equipment.[35] They use the resulting estimates to derive predictions for the magnitude of capital-skill complementarity in the U.S. economy. In particular, as the results presented in Section 2.3 show, the nested CES aggregate production function implies

$$\sigma_{\ell e} - \sigma_{he} = \tfrac{\varsigma-\rho}{\theta_e+\theta_h}, \tag{29}$$

$$\frac{d\log\left(W_h/W_\ell\right)}{d\log W_e} = -\tfrac{\theta_e(\varsigma-\rho)}{\theta_h\,\varsigma+\theta_e\,\rho}. \tag{30}$$

The first and second columns in Table 11 compare our baseline results on the strength of capital-skill complementarity in the French economy from Section 5.1 with those of Krusell et al. (2000). While both results are consistent with capital-skill complementarity at the aggregate level, the magnitude of our estimates is substantially lower.

We consider three potential explanations for the gap in the two sets of estimates. First, the weaker capital-skill complementarity that we estimate could be driven by our as-

[35]For more recent work revisiting their estimates and extending the data to more recent time periods, see, e.g., Castex et al. (2022), Maliar et al. (2022), and Ohanian et al. (2023).

Table 11: Comparison with the Nested CES Aggregate Production Function (KORV)

	Baseline France 1997-2007	KORV US 1963-1992	Our GMM approach US 1963-1992	Our GMM approach France 1997-2007
ς	–	1.67	1.50	0.91
ρ	–	0.67	0.71	0.39
$\sigma_{\ell e} - \sigma_{he}$	0.24	2.39	1.90	0.76
$\frac{d\log(W_h/W_\ell)}{d\log W_e}$	-0.07	-0.39	-0.33	-0.37

Notes: $\sigma_{\ell e} - \sigma_{he}$ is the aggregate degree of capital-skill complementarity and $d\log(W_h/W_\ell)/d\log W_e$ is the elasticity of the skill premium with respect to the equipment price. Column 1 is our baseline, column 2 displays the value for the elasticity estimated by Krusell et al. (2000) using U.S. data between 1963-1992, column 3 replicates column 2 but using the estimation approach we provide in Appendix C.7. Finally column 4 displays the value for the elasticities we estimate using French data over our sample years of 1997-2007. We use aggregate factor payments together with a quality-adjusted aggregate equipment price shock $\widetilde{W}_{et}$ and compositionally-adjusted aggregate average wages. We further rescale factor shares to match the labor share for the entire economy in 1997. Details of the data construction for France are provided in Appendix C.1.7 and C.2.3.

sumption that firm-level production functions are in the CRESH family. However, as we show in Table 9, we obtain very similar results using CXES and nested CES production functions at the firm level. Second, the weaker capital-skill complementarity that we estimate could be driven by our focus on France. To check if this is the case, we estimate the aggregate nested CES production function using French data. In Appendix C.7, we provide a strategy to estimate these aggregate parameters using simple log-linear estimation equations, based on the same data and the same identification assumptions as those of Krusell et al. (2000). As the second and third columns of Table 11 show, our strategy leads to similar estimates to those of Krusell et al. (2000) when using U.S. data. The fourth column of the table shows the results of applying this estimation strategy on aggregate French data over the 1997-2007 period used in our baseline estimation. We find estimates that are similar in magnitude to those of Krusell et al. (2000).[36] We conclude that the differences in results are neither driven by our assumption on firm-level production functions nor are they driven by differences between the two economies or the time periods under consideration.[37]

What then explains the gap between the two sets of estimates? Appendix A.5 shows

[36] And the estimate of the degree of aggregate capital-skill complementarity in France is also similar to the magnitude we calibrate heroically in Section 3.3.

[37] To better match aggregate data and the approach in Krusell et al. (2000), we implement two adjustments to our aggregated data. We rescale factor shares to match the aggregate labor share in 1997 and we apply a quality adjustment to our aggregate capital series to match the average price difference between the series used in Krusell et al. (2000) and the official NIPA data. These adjustments do not drive our results: in their absence we find an even larger aggregate elasticity of the skill premium.

how we can use our baseline estimates to infer the trend in skill-augmenting technical change not embodied in capital equipment. This trend corresponds to the gap between the observed path of the skill premium in data and our predicted path accounting for changes in the equipment price and skill supply in Figure 4a. The figure clearly shows that this gap grows substantially over the 1997-2007 period, indicating the presence of skill-augmenting technical change. The identification assumption of Krusell et al. (2000) rules out this trend. Hence, it overestimates the degree of capital-skill complementarity, attributing to this channel the entirety of the gap between the observed path of the skill premium and the predicted path accounting only for changes in skill supply (the dashed line in Figure 4a). Since our identification strategy relies on exogenous instruments at the firm level, we can avoid this bias by accommodating the presence of such potential trends in factor-augmenting productivity at the firm, industry, and aggregate levels.

This interpretation also clarifies what our results do and do not imply. Our estimates suggest that the rise in the relative demand for skill cannot be primarily explained by declines in the price of a broad notion of equipment capital. This conclusion does not imply that all capital-related technological change has small distributional effects. Narrower forms of capital, such as computers or robots, may affect relative factor demand differently from broad equipment capital. In our framework, improvements in their productivity or in the set of tasks they can perform may appear as factor-augmenting technological change rather than as capital-skill complementarity with respect to the price of broad equipment capital.

6 Conclusion

Does the aggregate production technology feature capital-skill complementarity? If so, to what extent? And is this aggregate capital-skill complementarity driven primarily by capital-skill complementarity at the firm level, or by a positive correlation between firm-level capital intensity and skill intensity?

To make progress on these questions, we characterized the response of the observed skill premium to changes in the price of equipment capital in a multi-factor, multi-sector framework featuring general, firm-specific, constant-returns-to-scale production functions. We showed how the aggregate elasticities of substitution that determine the degree of capital-skill complementarity depend on firm-level production and demand elasticities, as well as firm-level factor intensities. This contribution applies to any constant-returns-to-scale firm-level production function.

We then applied our theory to the French economy. To do so, we exploited several

French administrative data sources to measure firm-level factor intensities and wages for each skill group and to estimate the micro-level elasticities of substitution. We estimated statistically significant capital-skill complementarity at the firm level. Aggregating these estimates, we found that a 1% decline in the price of equipment generates an approximately 0.07% increase in the observed skill premium in France. This quantitative result is broadly robust to alternative firm-level production functions, heterogeneous equipment-price shocks across firms, and a more flexible demand system across firms.

Although we found a significant degree of capital-skill complementarity, our identification assumptions, and therefore our results, differ markedly from previous estimates obtained from aggregate time series. In particular, our empirical and theoretical strategies do not necessarily attribute the aggregate increase in the relative demand for skill *entirely* (or at all) to capital-skill complementarity. Our results also imply that capital-skill complementarity is not sufficiently strong to generate the full increase in the relative demand for high-skilled workers observed in the data. Explaining the evolution of the skill premium in France requires a sizable degree of skill-augmenting technical change not embodied in capital equipment. Of course, whereas our theoretical approach is general, we make no claims that our quantitative results apply more broadly than the French context in which we estimated elasticities and measured factor intensities. It remains an open question whether similar quantitative results on the aggregate degree of capital-skill complementarity obtain in other labor-market environments, such as the United States.

Appendix to "Capital-Skill Complementarity in Firms and in the Aggregate Economy"

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A Theoretical Appendix

A.1 Allen-Uzawa vs. Morishima Elasticities of Substitution

Define the Morishima elasticity of substitution between factors f and $f' \neq f$ as

$$\sigma^M_{ff'} \equiv \frac{\partial \log \left(X_f / X_{f'} \right)}{\partial \log W_{f'}}, \qquad f \neq f'.$$

Note that, unlike the Allen-Uzawa elasticities, the Morishima elasticities are not symmetric: $\sigma^M_{ff'} \neq \sigma^M_{f'f}$.

The gap between two Morishima elasticities—between factors f and $f'' \neq f$ and between factors f' and $f'' \neq f'$—is given by

$$\sigma^M_{ff''} - \sigma^M_{f'f''} = \frac{\partial \log \left(X_f / X_{f'} \right)}{\partial \log W_{f''}} = \theta_{f''} \left(\sigma_{ff''} - \sigma_{f'f''} \right), \qquad f \neq f' \neq f''.$$

This implies that the numerator of equation (6) satisfies:

$$\sigma^M_{\ell e} - \sigma^M_{he} = \theta_e \left(\sigma_{\ell e} - \sigma_{he} \right).$$

More generally, we can relate Morishima elasticities to the Allen-Uzawa elasticities defined in equation (3). Using equation (3) again, we can write the Morishima elasticity as

$$\begin{aligned} \sigma^M_{ff'} &= \frac{\partial \log X_f}{\partial \log W_{f'}} - \frac{\partial \log X_{f'}}{\partial \log W_{f'}}, \\ &= \theta_{f'} \sigma_{ff'} + \sum_{f'' \neq f'} \theta_{f''} \sigma_{f'f''}, \\ &= \left(1 - \sum_{f'' \notin \{f,f'\}} \theta_{f''} \right) \sigma_{ff'} + \left(\sum_{f'' \notin \{f,f'\}} \theta_{f''} \right) \sum_{f'' \notin \{f,f'\}} \frac{\theta_{f''}}{\sum_{f'' \notin \{f,f'\}} \theta_{f''}} \sigma_{f'f''}. \end{aligned}$$

The Morishima elasticity $\sigma^M_{ff'}$ is given by the convex combination of the Allen-Uzawa elasticity $\sigma_{ff'}$ and the factor-intensity weighted mean of $\sigma_{f'f''}$ within the set of factors

$f'' \in \mathcal{F} \setminus \{f, f'\}$, where the latter average is weighted in the linear combination by the cost share of all factors $\mathcal{F} \setminus \{f, f'\}$ in total costs. For instance, for the elasticity $\sigma^M_{\ell h}$, the set $\mathcal{F} \setminus \{\ell, h\} = \{e\}$ and we find the denominator of the expression of equation (6):

$$\sigma^M_{\ell h} = (1 - \theta_e)\, \sigma_{\ell h} + \theta_e \sigma_{he}.$$

Just like the Allen-Uzawa elasticities, in the case of the CES (ζ) production function we find $\sigma^M_{ff'} = \zeta$.

A.2 Functional Forms for Production Functions

A.2.1 Nested CES

Elasticities of Substitution. Solving for factor demand given the firm-level production function defined in equation (17), we find $X_{hi}/X_{ci} = Z_{hi}^{\rho-1}\, (W_{hi}/W_{ci})^{-\rho}$, $X_{\ell i}/Y_i = Z_{\ell i}^{\varsigma-1}\, (W_{\ell i}/P_i)^{-\varsigma}$, and $X_{ci}/Y_i = (W_{ci}/P_i)^{-\varsigma}$, where we have defined the price of the bundle of equipment and high-skilled labor as $W_{ci} = \left((W_{hi}/Z_{hi})^{1-\rho} + (W_{ei}/Z_{ei})^{1-\rho} \right)^{\frac{1}{1-\rho}}$ and the price of output as $P_i = \left((W_{\ell i}/Z_{\ell i})^{1-\varsigma} + W_{ci}^{1-\varsigma} \right)^{\frac{1}{1-\varsigma}}$. We can now compute the elasticities of substitution as

$$\begin{aligned}
\sigma_{\ell e,i} &= \frac{1}{\theta_{ei}} \frac{\partial \log X_{\ell i}}{\partial \log W_{ei}} = \frac{1}{\theta_{ei}} \left(\varsigma \frac{\partial \log P_i}{\partial \log W_{ei}} \right) = \varsigma, \\
\sigma_{\ell h,i} &= \frac{1}{\theta_{hi}} \frac{\partial \log X_{\ell i}}{\partial \log W_{hi}} = \frac{1}{\theta_{hi}} \left(\varsigma \frac{\partial \log P_i}{\partial \log W_{hi}} \right) = \varsigma, \\
\sigma_{he,i} &= \frac{1}{\theta_{ei}} \left(\frac{\partial \log (X_{hi}/X_{ci})}{\partial \log W_{ci}} \frac{\partial \log W_{ci}}{\partial \log W_{ei}} + \frac{\partial \log X_{ci}}{\partial \log W_{ei}} \right), \\
&= \frac{1}{\theta_{ei}} \left(\rho \frac{\partial \log W_{ci}}{\partial \log W_{ei}} - \varsigma \left(\frac{\partial \log W_{ci}}{\partial \log W_{ei}} - \frac{\partial \log P_i}{\partial \log W_{ei}} \right) \right), \\
&= \frac{1}{\theta_{ei}} \left((\rho - \varsigma) \frac{\theta_{ei}}{\theta_{ei} + \theta_{hi}} + \varsigma\, \theta_{ei} \right) = \varsigma - \frac{\varsigma - \rho}{\theta_{ei} + \theta_{hi}},
\end{aligned}$$

where we have substituted from the definitions of W_{ci} and X_{ci} above, and have used the equality $\frac{\partial \log W_{ci}}{\partial \log W_{ei}} = \left(\frac{W_{ei}/Z_{ei}}{W_{ci}} \right)^{1-\rho} = \frac{W_{ei} X_{ei}}{W_{ci} X_{ci}} = \frac{\theta_{ei}}{\theta_{ei} + \theta_{hi}}$.

Demand System. Consider a sequence of firm-specific factor prices $\boldsymbol{W}_{it}$, which the firm treats as exogenous, and optimal input choices $\boldsymbol{X}_{it}$ characterized by the nested CES factor demand system defined in equation (17). Relative factor demands at any time t satisfy

$$\frac{X_{\ell it}}{X_{cit}} = Z_{\ell it}^{\varsigma-1}\left(\frac{W_{\ell it}}{W_{cit}}\right)^{-\varsigma}, \qquad \frac{X_{hit}}{X_{cit}} = Z_{hit}^{\rho-1}\left(\frac{W_{hit}}{W_{cit}}\right)^{-\rho}, \tag{A.1}$$

Using these expressions, we relate the skill premium to firm i's factor input choices as

$$\frac{W_{hit}}{W_{\ell it}} = \frac{Z_{hit}^{\frac{\rho-1}{\rho}}}{Z_{\ell it}^{\frac{\varsigma-1}{\varsigma}}}\left(\frac{X_{\ell it}}{X_{hit}}\right)^{\frac{1}{\varsigma}}\left(\frac{X_{hit}}{X_{cit}}\right)^{\frac{1}{\varsigma}-\frac{1}{\rho}}. \tag{A.2}$$

Next, we use equation (A.1) again to write

$$\frac{\theta_{hit}}{\theta_{eit}+\theta_{hit}} = \frac{W_{hit}X_{hit}}{W_{cit}X_{cit}} = Z_{hit}^{\rho-1}\left(\frac{W_{hit}}{W_{cit}}\right)^{1-\rho},$$

which we use to substitute for X_{hit}/X_{cit} in equation (A.2) to find

$$\frac{W_{hit}}{W_{\ell it}} = Z_{hit}^{\left(\frac{1}{\rho}-\frac{1}{\varsigma}\right)}\frac{Z_{hit}^{\frac{\rho-1}{\rho}}}{Z_{\ell it}^{\frac{\varsigma-1}{\varsigma}}}\left(\frac{X_{\ell it}}{X_{hit}}\right)^{\frac{1}{\varsigma}}\left(\frac{\theta_{hit}}{\theta_{eit}+\theta_{hit}}\right)^{\frac{\rho}{1-\rho}\left(\frac{1}{\rho}-\frac{1}{\varsigma}\right)}.$$

Combining this equation with the remaining relative factor demand equation,

$$\frac{X_{hit}}{X_{eit}} = \left(\frac{Z_{hit}}{Z_{eit}}\right)^{\rho-1}\left(\frac{W_{hit}}{W_{eit}}\right)^{-\rho}$$

we obtain the following demand system

$$x_{hit}-x_{\ell it} = -\varsigma\,(w_{hit}-w_{\ell it}) + \frac{\varsigma-\rho}{1-\rho}\log\left(\frac{\theta_{hit}}{\theta_{eit}+\theta_{hit}}\right) + (\varsigma-1)\,(z_{hit}-z_{\ell it}), \tag{A.3}$$

$$x_{hit}-x_{eit} = -\rho\,(w_{hit}-w_{eit}) + (\rho-1)\,(z_{hit}-z_{eit}), \tag{A.4}$$

where $v \equiv \log V$ for any variable V.

A.2.2 CRESH

Elasticities of Substitution. We compute factor cost shares by minimizing unit costs:

$$\min_{(X_{fi})}\sum_f W_{fi}X_{fi} + \Xi_i\left(1 - \sum_{f\in\{\ell,h,e\}}\delta_f\left(\frac{Z_{fi}X_{fi}}{Y_i}\right)^{\frac{\sigma_f-1}{\sigma_f}}\right),$$

where Ξ_i is the Lagrange multiplier on the constraint defining the production function in equation (20). The first order condition with respect to factor f gives us:

$$W_{fi} = \Xi_i \, \delta_f \, \frac{\sigma_f - 1}{\sigma_f} \frac{1}{X_{fi}} \left(\frac{Z_{fi} X_{fi}}{Y_i} \right)^{\frac{\sigma_f - 1}{\sigma_f}},$$

implying $\theta_{fi} \propto W_{fi} X_{fi} \propto \delta_f \, \frac{\sigma_f - 1}{\sigma_f} \left(\frac{Z_{fi} X_{fi}}{Y_i} \right)^{\frac{\sigma_f - 1}{\sigma_f}}$.

It follows that the ratio of payments to factors f and f' satisfy:

$$\frac{W_{fi}}{W_{f'i}} = \frac{\delta_f \, \frac{\sigma_f - 1}{\sigma_f} \left(\frac{Z_{fi}}{Y_i} \right)^{\frac{\sigma_f - 1}{\sigma_f}} X_{fi}^{-\frac{1}{\sigma_f}}}{\delta_{f'} \, \frac{\sigma_{f'} - 1}{\sigma_{f'}} \left(\frac{Z_{f'i}}{Y_i} \right)^{\frac{\sigma_{f'} - 1}{\sigma_{f'}}} X_{f'i}^{-\frac{1}{\sigma_{f'}}}}. \tag{A.5}$$

It then follows that for any $f \neq f'$, we find the following condition on the price elasticities of factor demand

$$1 = \frac{1}{\sigma_f} \frac{\partial \log X_{fi}}{\partial \log W_{f'i}} - \frac{1}{\sigma_{f'}} \frac{\partial \log X_{f'i}}{\partial \log W_{f'i}}. \tag{A.6}$$

Using the constraint (20) again, we can find another constraint on the price elasticities of factor demand

$$0 = \frac{\partial}{\partial \log W_{f'i}} \left[\sum_{f \in \mathcal{F}} \delta_f \left(\frac{Z_{fi} X_{fi}}{Y_i} \right)^{\frac{\sigma_f - 1}{\sigma_f}} \right] = \sum_f \delta_f \left(\frac{\sigma_f - 1}{\sigma_f} \right) \left(\frac{Z_{fi} X_{fi}}{Y_i} \right)^{\frac{\sigma_f - 1}{\sigma_f}} \frac{\partial \log X_{fi}}{\partial \log W_{f'i}} = \sum_f \theta_{fi} \frac{\partial \log X_{fi}}{\partial \log W_{f'i}}, \tag{A.7}$$

where in the last equality we have used the result that $\theta_{fi} \propto \delta_f \, \frac{\sigma_f - 1}{\sigma_f} \left(\frac{Z_{fi} X_{fi}}{Y_i} \right)^{\frac{\sigma_f - 1}{\sigma_f}}$ proven above.

From equation (A.7), we find

$$\begin{aligned} 0 &= \theta_{f'i} \frac{\partial \log X_{f'i}}{\partial \log W_{f'i}} + \sum_{f \in \mathcal{F} / \{f'\}} \theta_{fi} \left(\sigma_f + \frac{\sigma_f}{\sigma_{f'}} \frac{\partial \log X_{f'i}}{\partial \log W_{f'i}} \right), \\ &= \left(\sum_{f \in \mathcal{F}} \theta_{fi} \sigma_f \right) \frac{\partial \log X_{f'i}}{\partial \log W_{f'i}} + \sigma_{f'} \left(\sum_{f \in \mathcal{F} / \{f'\}} \theta_{fi} \sigma_f \right), \\ &= \bar{\sigma}_i \left(\frac{\partial \log X_{f'i}}{\partial \log W_{f'i}} + \sigma_{f'} \left(1 - \frac{\theta_{f'i} \sigma_{f'}}{\bar{\sigma}_i} \right) \right), \end{aligned}$$

where in the first equality we have substituted for $\frac{\partial \log X_{fi}}{\partial \log W_{f'i}}$ from equation (A.6), where we have multiplied the expression by $\sigma_{f'}$ in the second equality, and where we have defined $\overline{\sigma}_i \equiv \sum_{f\in\mathcal{F}} \theta_{fi}\sigma_f$ in the third equality. It then follows that:

$$\frac{\partial \log X_{f'i}}{\partial \log W_{f'i}} = \frac{\sigma_{f'}^2}{\overline{\sigma}_i}\theta_{f'i} - \sigma_{f'}.$$

Substituting this expression in equation (A.6), we find

$$\begin{aligned}\frac{\partial \log X_{fi}}{\partial \log W_{f'i}} &= \sigma_f + \frac{\sigma_f}{\sigma_{f'}}\frac{\partial \log X_{f'i}}{\partial \log W_{f'i}},\\ &= \frac{\sigma_f \sigma_{f'}}{\overline{\sigma}_i}\theta_{f'i}.\end{aligned}$$

Equation (21) then follows from the definition of the elasticities of substitution.

Demand System. Consider a sequence of firm-specific factor prices $\boldsymbol{W}_{it}$, which the firm treats as exogenous, and optimal input choices $\boldsymbol{X}_{it}$ characterized by the CRESH factor demand system defined in equation (20). Rewriting equation (A.5) in logs, and denoting by $v \equiv \log V$ for any variable V, we find for $f \in \{\ell, h\}$:

$$\begin{aligned}w_{eit} - w_{fit} = \frac{1}{\sigma_f}x_{fit} - \frac{1}{\sigma_e}x_{eit} + \left(\frac{1}{\sigma_e} - \frac{1}{\sigma_f}\right)y_{it} + \left(1 - \frac{1}{\sigma_e}\right)z_{eit} - \left(1 - \frac{1}{\sigma_f}\right)z_{fit}\\ + \log\left(\delta_e \frac{\sigma_e - 1}{\sigma_e}\right) - \log\left(\delta_f \frac{\sigma_f - 1}{\sigma_f}\right).\end{aligned}$$

From this expression, the following demand system follows for $f \in \{\ell, h\}$

$$x_{eit} - x_{fit} = -\sigma_f\ (w_{eit} - w_{fit}) + \left(\frac{\sigma_f}{\sigma_e} - 1\right)(y_{it} - x_{eit}) + (1 - \sigma_f)\, z_{fit} + \left(\sigma_f - \frac{\sigma_f}{\sigma_e}\right) z_{eit} + \varpi_f, \tag{A.8}$$

where we have defined $\varpi_f \equiv \sigma_f \log\left(\delta_e \frac{\sigma_e - 1}{\sigma_e}\right) - \sigma_f \log\left(\delta_f \frac{\sigma_f - 1}{\sigma_f}\right)$.

A.3 Theoretical Extensions of the Baseline Setting

A.3.1 Elastic Relative Labor Supply

Following the proof of Proposition 1, it is straightforward to derive the response of the skill premium to small changes both in the price of equipment and the relative supply of skill as

$$d\log\left(\frac{W_h}{W_\ell}\right)=-\frac{\theta_e\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}d\log W_e-\frac{1}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}d\log\left(\frac{\overline{X}_h}{\overline{X}_\ell}\right). \tag{A.9}$$

The first term on the right-hand side captures the elasticity of the skill premium with respect to the price of equipment at fixed supplies of labor, given in equation (6), and the second term captures the reciprocal of the elasticity of relative skill demand with respect to the skill premium.

The following proposition generalizes our main result in Proposition 1 to the case with a relative supply of skilled workers that responds to the relative wages.

Proposition A.1. *Consider the setting in Proposition 1 but assume that the skill supplies $\overline{X}_\ell\left(W_\ell,W_h\right)$ and $\overline{X}_h\left(W_\ell,W_h\right)$ are homogeneous of degree-zero functions of wages, such that the elasticity of the relative skill supply with respect to the skill premium is ξ:*

$$\frac{\partial\log\overline{X}_h/\overline{X}_\ell}{\partial\log W_h/W_\ell}=\xi\geq 0.$$

Assume a small shock to the equipment supply $\overline{X}_e$. The effect on the skill premium, i.e., the relative prices of high-skilled and low-skilled labor, satisfies

$$\frac{d\log\left(W_h/W_\ell\right)}{d\log W_e}=-\frac{\theta_e\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}+\xi}, \tag{A.10}$$

where the aggregate elasticities of substitution $\sigma_{ff'}$ for $f,f'\in\mathcal{F}$ are defined by equation (3).

Proof. We use the fact that factor supply and factor demand equalize in equilibrium, $\overline{\boldsymbol{X}}=\boldsymbol{X}$. In this case, with the low-skilled wage as the numeraire $W_\ell\equiv 1$, the left hand side of equation (7) is given by $\frac{d\log\left(\overline{X}_h/\overline{X}_\ell\right)}{d\log W_e}=\frac{\partial\log\left(\overline{X}_h/\overline{X}_\ell\right)}{\partial\log W_h}\frac{d\log W_h}{d\log W_e}=\xi\frac{d\log W_h}{d\log W_e}$. Using the same additional steps as those in the proof of Proposition 1 leads to the desired result. □

A.3.2 Factor Price Heterogeneity

We begin this section with heterogeneous equipment prices, which generalize the results in Section 2.2 in the direction discussed in Section 2.4.2. We then consider a more general environment that features factor price heterogeneity across all factors, including unskilled and skilled labor.

Heterogeneity in Equipment Prices and the Response of the Skill Premium. Let $d\log W_{ei}=d\log T_{ei}+d\log W_e$ denote a small *exogenous* change in the log price of equipment faced by firm i, induced by underlying changes in the firm-specific wedge T_{ei} and the shadow price of equipment W_e. We take this vector of firm-level equipment-price changes as the primitive shock and define the corresponding change in the aggregate equipment price index as $d\log \overline{W}_e \equiv \sum_i \Lambda_i \widehat{\theta}_{ei}\, d\log W_{ei}$, where Λ_i is defined in equation (13) and $\widehat{\theta}_{ei}$ is defined in equation (14). Moreover, we define the corresponding *exposure* weights as $\omega_{ei} \equiv d\log W_{ei}/d\log \overline{W}_e$. These weights are a normalization of the firm-level shock, rather than an additional set of primitives, and measure the relative exposure of firm i to the aggregate equipment-price change. The exposure measure may take a negative value since firm-specific equipment prices may move in the opposite direction of the aggregate index.

In general, given the definitions above, we can study the gradient of aggregate factor demand with respect to each of the many firm-specific equipment prices. To create a parallel to the standard concept of the aggregate elasticity of substitution, we instead fix the exposure weights ω_{ei} across firms and take a directional derivative of aggregate factor demand by considering its comparative statics with respect to the average magnitude $d\log \overline{W}_e$ of the shock to equipment prices. In parallel to our definition of the aggregate elasticity of substitution (3), we can define the *heterogeneity-adjusted* aggregate elasticity of substitution capturing the effect of the shock to equipment prices on the relative demand and payments for factor f as

$$\sigma^{\omega}_{fe} \equiv \frac{1}{\theta_e} \frac{\partial \log X_f}{\partial \log \overline{W}_e}\bigg|_{(\omega_{ei})}, \tag{A.11}$$

where the superscript ω indicates the fact that the elasticity is defined with respect to a given set of firm-specific exposure weights (ω_{ei}). Note that the aggregate elasticity of substitution defined in equation (3) is a special case of the aggregate wedge elasticity defined in equation (A.11) corresponding to the case of $\omega_{ei} \equiv 1$.

Proposition A.2. *Consider a setting with three-factors, $\mathcal{F} \equiv \{\ell, h, e\}$, with exogenous factor supplies and with firm-specific factor prices. Assume small shocks to equipment factor prices characterized by $d\log \overline{W}_e$ for a given vector of exposure weights $\boldsymbol{\omega}_e \equiv (\omega_{ei})$. Then, the response of the skill premium to this collection of heterogeneous shocks in the price of equipment is given by*

$$d\log\left(\frac{W_h}{W_\ell}\right) = -\frac{\theta_e\left(\sigma^{\omega}_{\ell e} - \sigma^{\omega}_{he}\right)}{(1-\theta_e)\,\sigma_{\ell h} + \theta_e \sigma_{eh}} d\log \overline{W}_e, \tag{A.12}$$

where the heterogeneity-adjusted elasticity of substitution can be related to the micro-level elasticities of production and demand based on the following generalization of equation (15), using the *relative-intensity notation in equation (14):*

$$\sigma^{\omega}_{fe} \equiv \mathbb{E}_i\left[\widehat{\theta}_{fi}\widehat{\theta}_{ei}\sigma_{fe,i}\omega_{ei}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\widehat{\theta}_{fi},\widehat{\theta}_{ei}\omega_{ei}\right)\right] - \eta\,\mathbb{C}_s\left(\widehat{\theta}_{fs},\widehat{\theta}_{es}\omega_{es}\right). \tag{A.13}$$

where we have defined the mean sector-specific wedge shock ω_{fs} as

$$\omega_{fs} \equiv \frac{\sum_{i\in\mathcal{I}_s}\Lambda_{i|s}\theta_{fi}\omega_{fi}}{\sum_{i\in\mathcal{I}_s}\Lambda_{i|s}\theta_{fi}}, \tag{A.14}$$

as the mean of the exposure weights based on the distribution of the share of sector-s payments to factor f paid by firm i.

Proof. This result is a special case of the results of Propositions A.3 and A.4 below for the case in which labor wages are uniform across firms. □

General Heterogeneity-Adjusted Elasticities of Substitution Let us now assume that firms face potential wedges in *all* factor markets and pay firm-and-factor-specific prices. We assume a fixed and exogenous wedge T_{fi} for each firm i and factor f. The firm-level factor price is $W_{fi} = W_f T_{fi}$, where we refer to W_f as the shadow price (or wage) of factor f, since it is unobservable in the data. The shadow factor price is common across firms and equates supply and demand, $X_f = \sum_i X_{fi}$. We take the shadow wage of low-skilled workers as the numeraire and normalize $W_\ell = 1$. In Appendix A.6, we present a micro-foundation for the labor-market wedges based on firm-and-labor-type specific compensating differentials in competitive factor markets (the appendix also offers a micro-foundation for firm-level heterogeneity in the price of equipment).

At the micro-level, we can express the elasticity of the factor intensity of factor f with respect to the price of factor f' in terms of the elasticity of substitution from equation (11) as

$$\frac{\partial \log \theta_{fi}}{\partial \log W_{f'}} = \frac{\partial \log\left(W_{fi}X_{fi}/\left(Y_iC_i\right)\right)}{\partial \log W_{f'}} = \theta_{f'i}\left(\sigma_{ff',i}-1\right).$$

Moving to the aggregate case, in the presence of factor price heterogeneity across firms, in general we have $\theta_f \neq W_f X_f/\left(YC\right)$, thus preventing us from writing a direct macro counterpart to the above equation. Instead, we may define a distinct measure of substitutability across factors in terms of factor payments. Let $\sigma^*_{ff'}$ for $f' \neq f$ denote the aggregate elasticity of factor payment substitution as

$$\sigma^*_{ff'} \equiv 1 + \frac{1}{\theta_{f'}} \frac{\partial \log \theta_f}{\partial \log W_{f'}}, \tag{A.15}$$

where $\theta_f \equiv \left(\sum_i W_{fi} X_{fi}\right) / \left(\sum_{f',i'} W_{f'i'} X_{f'i'}\right)$ denotes the aggregate factor-f intensity of all factor payments in the economy. When there are no wedges ($T_{fi} \equiv T_f$ for all i), the two aggregate elasticities of substitution defined in equations (3) and (A.15) coincide, $\sigma_{ff'} \equiv \sigma^*_{ff'}$. More generally, however, the two elasticities may diverge due to the fact that reallocations across firms have distinct effects on aggregate factor demand and factor payments.

In parallel to the discussion in Section 2.4.2, we can also consider heterogeneous shocks to factor prices. Let $d \log W_{fi}$ denote a small change in the log price of factor f faced by firm i, and, using the relative-intensity notation in equation (14), define $d \log \overline{W}_f \equiv \sum_i \Lambda_i \widehat{\theta}_{fi} \, d \log W_{fi}$ where Λ_i is defined in equation (13), and exposure weights ω_{fi} satisfying $d \log W_{fi} = \omega_{fi} \, d \log \overline{W}_f$. As in the case of equipment price changes, we fix the shares ω_{fi} across firms and consider the comparative statics with respect to the average magnitude $d \log \overline{W}_f$ of the shock in the price of factor f. In parallel to our definition of the aggregate elasticities of substitution (3) and (A.15), we can define the heterogeneity-adjusted aggregate elasticities capturing the effect of the firm-specific shocks to factor-f' prices on the relative demand and payments for factor f as

$$\sigma^{\omega}_{ff'} \equiv \frac{1}{\theta_{f'}} \frac{\partial \log X_f}{\partial \log \overline{W}_{f'}} \bigg|_{\left(\omega_{f'i}\right)}, \qquad \sigma^{*,\omega}_{ff'} \equiv 1 + \frac{1}{\theta_{f'}} \frac{\partial \log \theta_f}{\partial \log \overline{W}_{f'}} \bigg|_{\left(\omega_{f'i}\right)}, \tag{A.16}$$

where the superscript ω indicates the fact that the elasticity is defined with respect to a given set of firm-specific weights $(\omega_{f'i})$. Note that the aggregate elasticities of substitution defined in equations (3) and (A.15) are special cases of the aggregate wedge elasticity defined in equation (A.16) corresponding to the case of $\omega_{fi} \equiv 1$. The next proposition characterizes the aggregate wedge elasticities in their general form.

Proposition A.3. *The aggregate heterogeneity-adjusted elasticities of substitution $\sigma^{\omega}_{ff'}$ and $\sigma^{*,\omega}_{ff'}$ between factors f and f' defined by equations (A.16) are given by*

$$\sigma^{\omega}_{ff'} = \mathbb{E}_i \left[\frac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi} \widehat{\theta}_{f'i} \omega_{f'i} \sigma_{ff',i} \right] - \varepsilon \, \mathbb{E}_s \left[\mathbb{C}_{i|s} \left(\frac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi}, \widehat{\theta}_{f'i} \omega_{f'i} \right) \right] - \eta \, \mathbb{C}_s \left(\frac{\overline{W}_f}{W_{fs}} \widehat{\theta}_{fs}, \widehat{\theta}_{f's} \omega_{f's} \right), \tag{A.17}$$

$$\sigma_{ff'}^{*,\omega} = \mathbb{E}_i\left[\widehat{\theta}_{fi}\widehat{\theta}_{f'i}\omega_{f'i}\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\widehat{\theta}_{fi},\widehat{\theta}_{f'i}\omega_{f'i}\right)\right] - \eta\,\mathbb{C}_s\left(\widehat{\theta}_{fs},\widehat{\theta}_{f's}\omega_{f's}\right) \tag{A.18}$$

where we have defined the mean sector-specific wedge shock ω_{fs} in equation (A.14).

Proof. See Appendix A.4 on page A20. □

It is straightforward to see that in the cases where factor price shocks are uniform across firms, the above results simplify to

$$\sigma_{ff'} = \mathbb{E}_i\left[\frac{\overline{W}_f}{W_{fi}}\widehat{\theta}_{fi}\widehat{\theta}_{f'i}\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\frac{\overline{W}_f}{W_{fi}}\widehat{\theta}_{fi},\widehat{\theta}_{f'i}\right)\right] - \eta\,\mathbb{C}_s\left(\frac{\overline{W}_f}{W_{fs}}\widehat{\theta}_{fs},\widehat{\theta}_{f's}\right), \tag{A.19}$$

$$\sigma_{ff'}^{*} = \mathbb{E}_i\left[\widehat{\theta}_{fi}\widehat{\theta}_{f'i}\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\widehat{\theta}_{fi},\widehat{\theta}_{f'i}\right)\right] - \eta\,\mathbb{C}_s\left(\widehat{\theta}_{fs},\widehat{\theta}_{f's}\right). \tag{A.20}$$

In the presence of exogenous distortions, equations (A.19) and (A.20) decompose the two aggregate elasticities of substitution into three components: the first term accounts for the appropriate weighted average of the within-firm elasticities of substitution across firms, the second term accounts for the effect of within-sector reallocations across firms, and the last term accounts for the effect of cross-sectoral reallocations. In the case of the aggregate elasticity of substitution for demand (payments), the averages across firms are weighted by their shares in aggregate demand for (payment to) factors.

The Response of the Skill Premium with Heterogeneous Wages In the presence of wage heterogeneity, we define the *observed skill premium* Ψ as the average wage of high-skilled workers relative to the average wage of low-skilled workers,

$$\Psi \equiv \frac{\overline{W}_h}{\overline{W}_\ell} = \frac{\sum_i W_{hi}X_{hi}/X_h}{\sum_{i'} W_{\ell i'}X_{\ell i'}/X_\ell}, \tag{A.21}$$

where $X_\ell \equiv \sum_i X_{\ell i}$ and $X_h \equiv \sum_i X_{hi}$ are the aggregate demands for high- and low-skilled labor, respectively. We aim to characterize the response of the skill premium to a shock to the prices of different factors, particularly equipment prices. In our model, such changes in equipment prices can stem from changes in the aggregate supply of equipment goods X_e or from changes in firm-specific wedges T_{ei}.

Proposition A.4. *Consider a setting with three-factors, $\mathcal{F} \equiv \{\ell, h, e\}$, with exogenous factor supplies and with firm-specific factor prices. Assume small shocks to the equipment factor prices characterized by $d\log\overline{W}_e$ and the vector of exposure weights $\boldsymbol{\omega}_e \equiv (\omega_{ei})$. Then, the responses of* *the shadow and observed skill premium to this collection of heterogeneous shocks in the price of equipment are given by*

$$d\log W_h = -\frac{\theta_e\left(\sigma^{\omega}_{\ell e}-\sigma^{\omega}_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{eh}}d\log \overline{W}_e, \tag{A.22}$$

$$\begin{aligned} d\log\Psi &= -\frac{\theta_e\left(\sigma^{\omega}_{\ell e}-\sigma^{\omega}_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}d\log \overline{W}_e \\ &\quad + \left[\left(1-\theta_e\right)\sigma^{*}_{\ell h}+\theta_e\sigma^{*}_{eh}\right]\left(\frac{\theta_e\left(\sigma^{\omega}_{\ell e}-\sigma^{\omega}_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}-\frac{\theta_e\left(\sigma^{*,\omega}_{\ell e}-\sigma^{*,\omega}_{he}\right)}{\left(1-\theta_e\right)\sigma^{*}_{\ell h}+\theta_e\sigma^{*}_{he}}\right)d\log \overline{W}_e. \end{aligned} \tag{A.23}$$

Proof. See Appendix A.4 on page A21. □

In the special case where the equipment price changes are uniform, e.g., driven by a small shock to the equipment factor prices characterized by supply $\overline{X}_e$ that leaves the supplies of other factors intact, we can specialize the above results as above. The effect on the shadow skill premium $\frac{d\log W_h}{d\log W_e}$, i.e., the relative shadow prices of high-skilled and low-skilled labor, satisfies equation (6), and the effect on the observed skill premium is given by

$$\frac{d\log\Psi}{d\log W_e} = \frac{d\log W_h}{d\log W_e} - \left[\left(1-\theta_e\right)\sigma^{*}_{\ell h}+\theta_e\sigma^{*}_{eh}\right]\left(\frac{d\log W_h}{d\log W_e}+\frac{\theta_e\left(\sigma^{*}_{\ell e}-\sigma^{*}_{he}\right)}{\left(1-\theta_e\right)\sigma^{*}_{\ell h}+\theta_e\sigma^{*}_{he}}\right), \tag{A.24}$$

where the aggregate elasticities of substitution $\sigma_{ff'}$ and $\sigma^{*}_{ff'}$ for $f \neq f' \in \mathcal{F}$ are defined by equations (3) and (A.15), respectively. Comparing equations (A.24) and (6), we see that the gap between the observed and the shadow skill premia exists to the extent that the aggregate elasticities in terms of factor demand and factor payments deviate from one another, that is, $\frac{\theta_e\left(\sigma^{*}_{\ell e}-\sigma^{*}_{he}\right)}{(1-\theta_e)\sigma^{*}_{\ell h}+\theta_e\sigma^{*}_{he}} \neq \frac{\theta_e(\sigma_{\ell e}-\sigma_{he})}{(1-\theta_e)\sigma_{\ell h}+\theta_e\sigma_{he}}$.

A.3.3 Composite Factors of Production: the Case of Labor Share

We can use our results to derive the elasticities of substitution for composite factors that are bundles of other factors. Let factor c be defined as a composite of a set $\mathcal{F}_c$ of other factors. For instance, we typically define labor (which we denote with label n) as the sum of all workers irrespective of their skill type such that $X_n \equiv X_\ell + X_h$ where we have $\mathcal{F}_n \equiv \{\ell, h\}$. We can show that the aggregate elasticity of factor payments substitution in equation (A.15), when defined between a composite factor c and any other factor f' that is not within the composite factor ($f' \notin \mathcal{F}_c$), is simply the convex combination of all the factors constituting the composite, that is,

$$\sigma^*_{cf'} \equiv 1 + \frac{1}{\theta_{f'}} \frac{\partial \log \theta_c}{\partial \log W_{f'}} = \sum_{f \in \mathcal{F}_c} \frac{\theta_f}{\theta_c} \sigma^*_{ff'}, \qquad f' \notin \mathcal{F}_c, \tag{A.25}$$

where the intensity of the composite factor is sum of the intensities of all the factors included in it, $\theta_c \equiv \sum_{f \in \mathcal{F}_c} \theta_f$, and where $\sigma^*_{ff'}$ is the aggregate elasticity of factor payments between f and f', given by equation (A.20). In the example of labor as the composite factor, the aggregate elasticity of substitution between labor and equipment is a convex combination of the aggregate elasticities of the factor payments for the two skill groups, with each group's weight given by the share of that skill group in labor payments, that is, $\sigma^*_{ne} = \frac{\theta_\ell}{\theta_n}\sigma^*_{\ell e} + \frac{\theta_h}{\theta_n}\sigma^*_{he}$.

The following result characterizes the elasticity of the aggregate share of a composite factor c with respect to the price of factor f'.

Proposition A.5. *The elasticity of the aggregate intensity of a composite factor c relative to the price of a given factor f' is given by*

$$\frac{\partial \log \theta_c}{\partial \log W_{f'}} = \begin{cases} \theta_{f'}\left(\sigma^*_{cf'} - 1\right), & f' \notin \mathcal{F}_c, \\ \theta_{f'}\left[\frac{\theta_{\bar{c}_{f'}}}{\theta_c}\left(\sigma^*_{\bar{c}_{f'}f'} - 1\right) - \frac{\theta_{\bar{f}'}}{\theta_c}\left(\sigma^*_{\bar{f}'f'} - 1\right)\right], & f' \in \mathcal{F}_c, \end{cases} \tag{A.26}$$

where we have defined two composite factors: a composite factor $\bar{f}$ as the set of all factors other than f (with $\mathcal{F}_{\bar{f}} \equiv \mathcal{F} / \{f\}$) and composite factor $\bar{c}_f$ as the set of all factors in the composite factor c other than f (with $\mathcal{F}_{\bar{c}_f} \equiv \mathcal{F}_c / \{f\}$).

Proof. See Appendix A.4 on page A23. □

In the example of the composite labor factor n, and in a three-factor model with $\mathcal{F} = \{\ell, h, e\}$, equation (A.26) implies that the elasticity of the labor share with respect to the price of equipment and the wage of high-skilled workers are given by $\frac{\partial \log \theta_n}{\partial \log W_e} = \theta_e\left(\sigma^*_{ne} - 1\right)$ (applying the first case of equation (A.26), since $e \notin \mathcal{F}_n$), and $\frac{\partial \log \theta_n}{\partial \log W_h} = \frac{\theta_e \theta_h}{\theta_n}\left(1 - \sigma^*_{eh}\right)$ (applying the second case, since $h \in \mathcal{F}_n$, with $\mathcal{F}_{\bar{n}_h} = \{\ell\}$ and $\mathcal{F}_{\bar{h}} = \{\ell, e\}$).[A1]

[A1] See the derivation, along with the proof for Proposition A.5, in Appendix A.4 on page A23.

A.3.4 Additional Factors of Production

To state our main result, we express the aggregate elasticities of substitution in vector and matrix form, defining the matrix $\mathbf{\Sigma}$, the vector of factor intensities $\boldsymbol{\theta}$, and the vectors $\boldsymbol{\sigma}_{\cdot f'}$ as

$$\mathbf{\Sigma} \equiv \left[\sigma_{ff'}\right]_{f,f' \in \mathcal{F}}, \qquad \boldsymbol{\theta} \equiv \left(\theta_\ell, \theta_h, \cdots, \theta_e\right)', \qquad \boldsymbol{\sigma}_{\cdot f'} \equiv \left(\sigma_{ff'}\right)_{f \in \mathcal{F}}, \tag{A.27}$$

and where the aggregate substitution elasticity of factor demand $\sigma_{ff'}$ is given by equation (3), with the additional extension $\sigma_{ff} \equiv 0$ for all f. Using these definitions, the following proposition characterizes the response.

Proposition A.6. *Consider a small shock to the supply of equipment that shifts the price of equipment, holding all other factor supplies and wedges constant. Let $\widetilde{\boldsymbol{W}} \equiv \left(W_f\right)_{f \in \mathcal{F} \setminus \{\ell, e\}}$ be the vector of the prices of all factors other than equipment and unskilled labor. The elasticity of this vector with respect to the price of capital equipment is given by*

$$\frac{d \log \widetilde{\boldsymbol{W}}}{d \log W_e} = -\theta_e \left(\widetilde{\boldsymbol{P}}_R' \, \boldsymbol{D} \, \widetilde{\boldsymbol{P}}_L\right)^{-1} \left(\sigma_{\ell e} - \widetilde{\boldsymbol{\sigma}}_e\right), \tag{A.28}$$

where we have defined $\widetilde{\boldsymbol{\sigma}}_e \equiv \left(\sigma_{fe}\right)_{f \in \mathcal{F} \setminus \{\ell, e\}}$, the matrix $\boldsymbol{D}$ is the demand elasticity matrix defined for the matrix of elasticities of substitution and the vector of factor intensities in equation (A.27) as

$$\boldsymbol{D} = diag\left(\mathbf{\Sigma}\, \boldsymbol{\theta}\right) - \mathbf{\Sigma}\, diag\left(\boldsymbol{\theta}\right), \tag{A.29}$$

and where the projection matrices $\widetilde{\boldsymbol{P}}_R$ and $\widetilde{\boldsymbol{P}}_L$ are $N \times (N-2)$ dimensional matrices defined as

$$\widetilde{\boldsymbol{P}}_R \equiv \begin{bmatrix} -1 & -1 & \cdots & -1 \\ 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \qquad \widetilde{\boldsymbol{P}}_L \equiv \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{A.30}$$

Proof. See page A18 in Appendix A.4. □

It is straightforward to show that the above results lead to Proposition 1 in the special case of $N = 3$. In this case, we have

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0 & \sigma_{\ell h} & \sigma_{\ell e} \\ \sigma_{\ell h} & 0 & \sigma_{he} \\ \sigma_{\ell e} & \sigma_{he} & 0 \end{bmatrix}, \qquad \boldsymbol{D} = \begin{bmatrix} \theta_h\sigma_{\ell h} + \theta_e\sigma_{\ell e} & -\theta_h\sigma_{\ell h} & -\theta_e\sigma_{\ell e} \\ -\theta_\ell\sigma_{\ell h} & \theta_\ell\sigma_{\ell h} + \theta_e\sigma_{he} & -\theta_e\sigma_{he} \\ -\theta_\ell\sigma_{\ell e} & -\theta_h\sigma_{he} & \theta_\ell\sigma_{\ell e} + \theta_h\sigma_{he} \end{bmatrix},$$

and we also have

$$\widetilde{\boldsymbol{P}}_R \equiv \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \qquad \widetilde{\boldsymbol{P}}_L \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

which leads to

$$\widetilde{\boldsymbol{P}}_R' \, \boldsymbol{D} \, \widetilde{\boldsymbol{P}}_L = \theta_h\sigma_{\ell h} + \theta_\ell\sigma_{\ell h} + \theta_e\sigma_{he} = (1-\theta_e)\,\sigma_{\ell h} + \theta_e\sigma_{he}.$$

Noting $\widetilde{\boldsymbol{W}} = W_h$ and $\widetilde{\boldsymbol{\sigma}}_e = \sigma_{he}$, we find the desired result in equation (6).

A.4 Proofs and Derivations

A.4.1 Proofs

Proof for Proposition 2. Let C_i, C_s, and C stand for the firm-level, sector-level, and aggregate unit cost of production, respectively. To compute the aggregate elasticities of substitution defined in equations (3), and using the relative-intensity notation in equation (14), first note that

$$\begin{aligned} \frac{\partial \log X_f}{\partial \log W_{f'}} &= \frac{1}{X_f} \sum_i X_{fi} \frac{\partial \log X_{fi}}{\partial \log W_{f'}}, \\ &= \sum_i \frac{X_{fi}}{X_f} \left[\frac{\partial \log \left(X_{fi}/Y_i \right)}{\partial \log W_{f'}} + \frac{\partial \log Y_i}{\partial \log W_{f'}} \right], && \text{(A.31)} \\ &= \sum_i \Lambda_i \widehat{\theta}_{fi} \left(\theta_{f'i}\sigma_{ff',i} + \frac{\partial \log Y_i}{\partial \log W_{f'}} \right), && \text{(A.32)} \end{aligned}$$

where in the last equality, we have used the fact that the production function is CRS and that the elasticity of substitution is defined for a constant scale of output, and the fact that $X_{fi}/X_f = Y_i C_i / (YC) \times (W_f X_{fi} / (Y_i C_i)) / (W_f X_f / (YC)) = \Lambda_i \widehat{\theta}_{fi}$. Crucially, note that the term $\frac{\partial \log Y_i}{\partial \log W_{f'}}$ captures the reallocations of production Y_i due to the response of firm-level demand in the product markets.

With monopolistic competition and CES demand in equation (1) and cost minimization, we obtain

$$\frac{\partial \log Y_i}{\partial \log W_{f'}} = -\varepsilon\left(\frac{\partial \log C_i}{\partial \log W_{f'}} - \frac{\partial \log C_s}{\partial \log W_{f'}}\right) - \eta\left(\frac{\partial \log C_s}{\partial \log W_{f'}} - \frac{\partial \log C}{\partial \log W_{f'}}\right),$$
$$= -\varepsilon\left(\theta_{f'i} - \theta_{f's}\right) - \eta\left(\theta_{f's} - \theta_{f'}\right), \tag{A.33}$$

where we have kept the scale of aggregate output Y as fixed, and where in the second equality, we have used Shephard's lemma, given the fact that the allocation of factors across firms within sectors is efficient. Substituting equation (A.33) in equation (A.32), we find

$$\sigma_{ff'} = \frac{1}{\theta_f\theta_{f'}}\left\{\sum_i \Lambda_i\theta_{fi}\theta_{f'i}\sigma_{ff',i} - \varepsilon\sum_i \Lambda_i\theta_{fi}\left(\theta_{f'i} - \theta_{f's}\right) - \eta\sum_i \Lambda_i\theta_{fi}\left(\theta_{f's} - \theta_{f'}\right)\right\},$$
$$= \frac{1}{\theta_f\theta_{f'}}\left\{\sum_i \Lambda_i\theta_{fi}\theta_{f'i}\sigma_{ff',i} - \varepsilon\sum_s \Lambda_s\sum_{i\in\mathcal{I}_s} \Lambda_{i|s}\theta_{fi}\left(\theta_{f'i} - \theta_{f's}\right) - \eta\sum_s \Lambda_s\sum_{i\in\mathcal{I}_s} \Lambda_{i|s}\theta_{fi}\left(\theta_{f's} - \theta_{f'}\right)\right\},$$
$$= \frac{1}{\theta_f\theta_{f'}}\left\{\mathbb{E}_i\left[\theta_{fi}\theta_{f'i}\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\theta_{fi},\theta_{f'i}\right)\right] - \eta\,\mathbb{C}_s\left(\theta_{fs},\theta_{f's}\right)\right\},$$
$$= \mathbb{E}_i\left[\widehat{\theta}_{fi}\widehat{\theta}_{f'i}\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\widehat{\theta}_{fi},\widehat{\theta}_{f'i}\right)\right] - \eta\,\mathbb{C}_s\left(\widehat{\theta}_{fs},\widehat{\theta}_{f's}\right),$$

where we have defined the expected value and covariance operators $\mathbb{E}_i(\cdot)$ and $\mathbb{C}_i(\cdot,\cdot)$, respectively, across firms i with a distribution implied by the shares of firms in aggregate payments Λ_i. Similarly, the same operators are defined across sectors $\mathbb{E}_s(\cdot)$ and $\mathbb{C}_s(\cdot,\cdot)$, with the corresponding shares across sectors Λ_s, and across firms within a given sector s as $\mathbb{E}_{i|s}(\cdot)$ and $\mathbb{C}_{i|s}(\cdot,\cdot)$, with the corresponding shares across sectors $\Lambda_{i|s} \equiv \Lambda_i/\Lambda_s$. □

Proof of equation (9) and (22). From equation (8), we have the following condition for a fall in the real wages of low-skilled workers:

$$1 < \frac{\theta_h\left(\sigma_{\ell e} - \sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h} + \theta_e\sigma_{he}},$$
$$= \frac{\theta_h\left(\sigma_\ell - \sigma_h\right)}{\left(1-\theta_e\right)\sigma_\ell + \theta_e\sigma_e}\frac{\sigma_e}{\sigma_h},$$

where equation (9) is immediate from the first line and we have substituted for $\sigma_{ff'}$ from equation (21) in the second line. This simplifies to

$$\frac{\sigma_\ell}{\sigma_h} - 1 > \frac{\left(1-\theta_e\right)\sigma_\ell + \theta_e\sigma_e}{\theta_h\sigma_e},$$

$$
\begin{aligned}
&= \frac{1}{\theta_h \sigma_e} \left(\sigma_e + (1-\theta_e)(\sigma_\ell - \sigma_e)\right), \\
&= \frac{1}{\theta_h} \left(\theta_e + (1-\theta_e) \frac{\sigma_\ell}{\sigma_e}\right),
\end{aligned}
$$

leading to the desired result. □

Proof of equation (19). Substituting the expressions in equation (18) in equation (8), we find

$$
\begin{aligned}
\frac{d \log (W_\ell / P)}{d \log W_e} &= -\theta_e \left(1 - \frac{\theta_h (\sigma_{\ell e} - \sigma_{he})}{(1-\theta_e)\sigma_{\ell h} + \theta_e \sigma_{he}}\right), \\
&= -\theta_e \left(1 - \frac{\frac{\theta_h}{\theta_e+\theta_h}(\varsigma - \rho)}{\varsigma - \frac{\theta_e}{\theta_e+\theta_h}(\varsigma - \rho)}\right), \\
&= -\theta_e \left(\frac{\varsigma - \frac{\theta_e}{\theta_e+\theta_h}(\varsigma-\rho) - \frac{\theta_h}{\theta_e+\theta_h}(\varsigma-\rho)}{\varsigma - \frac{\theta_e}{\theta_e+\theta_h}(\varsigma-\rho)}\right), \\
&= -\theta_e \frac{\rho}{\frac{\theta_h}{\theta_e+\theta_h}\varsigma + \frac{\theta_e}{\theta_e+\theta_h}\rho},
\end{aligned}
$$

reaching the desired result. □

Proof for equation (16). Using equation (15) and the relative-intensity notation in equation (14), we have

$$
\begin{aligned}
\sigma_{\ell e} - \sigma_{he} &= \mathbb{E}_i \left[\left(\widehat{\theta}_{\ell i}\sigma_{\ell e,i} - \widehat{\theta}_{hi}\sigma_{he,i}\right)\widehat{\theta}_{ei}\right] \\
&\quad + \varepsilon\, \mathbb{E}_s \left[\mathbb{C}_{i|s}\left(\widehat{\theta}_{hi} - \widehat{\theta}_{\ell i}, \widehat{\theta}_{ei}\right)\right] + \eta\, \mathbb{C}_s \left(\widehat{\theta}_{hs} - \widehat{\theta}_{\ell s}, \widehat{\theta}_{es}\right), \\
&= \mathbb{E}_i \left[\left(\frac{1}{2}\widehat{\theta}_{\ell i} + \frac{1}{2}\widehat{\theta}_{hi}\right)(\sigma_{\ell e,i} - \sigma_{he,i})\widehat{\theta}_{ei}\right] - \mathbb{E}_i \left[\left(\widehat{\theta}_{hi} - \widehat{\theta}_{\ell i}\right)\left(\frac{1}{2}\sigma_{\ell e,i} + \frac{1}{2}\sigma_{he,i}\right)\widehat{\theta}_{ei}\right] \\
&\quad + \varepsilon\, \mathbb{E}_s \left[\mathbb{C}_{i|s}\left(\widehat{\theta}_{hi} - \widehat{\theta}_{\ell i}, \widehat{\theta}_{ei}\right)\right] + \eta\, \mathbb{C}_s \left(\widehat{\theta}_{hs} - \widehat{\theta}_{\ell s}, \widehat{\theta}_{es}\right),
\end{aligned}
$$

where in the second equality we have expanded the term $\widehat{\theta}_{\ell i}\sigma_{\ell e,i} - \widehat{\theta}_{hi}\sigma_{he,i}$. To obtain the displayed form in equation (16), define

$$
a_i \equiv \frac{1}{2}\widehat{\theta}_{\ell i} + \frac{1}{2}\widehat{\theta}_{hi}, \qquad b_i \equiv \sigma_{\ell e,i} - \sigma_{he,i}, \qquad c_i \equiv \widehat{\theta}_{hi} - \widehat{\theta}_{\ell i}, \qquad d_i \equiv \frac{1}{2}\sigma_{\ell e,i} + \frac{1}{2}\sigma_{he,i},
$$

Since $\mathbb{E}^e_i[g_i] \equiv \mathbb{E}_i\left[\widehat{\theta}_{ei} g_i\right]$, the two within-firm terms can be written as

$$
\mathbb{E}_i\left[\widehat{\theta}_{ei} a_i b_i\right] = \mathbb{E}^e_i[a_i]\,\mathbb{E}^e_i[b_i] + \mathbb{C}^e_i(a_i, b_i),
$$

$$\mathbb{E}_i\left[\widehat{\theta}_{ei}c_i d_i\right] = \mathbb{E}_i^e\left[c_i\right]\mathbb{E}_i^e\left[d_i\right] + \mathbb{C}_i^e\left(c_i, d_i\right).$$

Moreover, $\mathbb{E}_i\left[c_i\right] = 0$ and $\mathbb{E}_i\left[\widehat{\theta}_{ei}\right] = 1$, so $\mathbb{E}_i^e\left[c_i\right] = \mathbb{C}_i\left(c_i, \widehat{\theta}_{ei}\right)$. Substituting these identities yields equation (16). □

Proof for equations (26) and (27). Writing equation (A.8) from Appendix A.2.2 in first differences for each firm i, we find for $f \in \{\ell, h\}$:

$$\Delta x_{ei,t} - \Delta x_{fi,t} = -\sigma_f\left(\Delta w_{et} - \Delta w_{ft}\right) + \left(\frac{\sigma_f}{\sigma_e} - 1\right)\left(\Delta y_{it} - \Delta x_{ei,t}\right) + \left(1 - \sigma_f\right)\Delta z_{fi,t} + \sigma_f\left(1 - \frac{1}{\sigma_e}\right)\Delta z_{ei,t}. \tag{A.34}$$

Using the demand equation (25), we find the following relationship between the change in output and the change in revenues:

$$\begin{aligned}
\Delta y_{it} &= -\varepsilon\,\Delta p_{it} + (\varepsilon - \eta)\,\Delta p_{st} + \alpha_t + \Delta\phi_{it} \\
&= \frac{\varepsilon}{\varepsilon - 1}\left(\Delta r_{it} - (\varepsilon - \eta)\,\Delta p_{st} - \alpha_t - \Delta\phi_{it}\right) + (\varepsilon - \eta)\,\Delta p_{st} + \alpha_t + \Delta\phi_{it}, \\
&= \frac{\varepsilon}{\varepsilon - 1}\Delta r_{it} - \frac{\varepsilon - \eta}{\varepsilon - 1}\Delta p_{st} - \frac{1}{\varepsilon - 1}\left(\alpha_t + \Delta\phi_{it}\right).
\end{aligned}$$

Substituting the above result in equation (A.34) leads to the desired result. □

Proof for equations (C.8) and (C.9). First-differencing the nested-CES factor-demand equations (A.3) and (A.4) gives equations (C.8) and (C.9), respectively. The residuals $u_{\ell it}$ and u_{eit} collect the changes in firm-specific factor-augmenting productivities. □

Proof for Proposition A.6. The changes in aggregate factor demand and factor prices satisfy

$$\begin{aligned}
d\log X_f &= \sum_{f'}\frac{\partial\log X_f}{\partial\log W_{f'}}d\log W_{f'}, \\
&= \frac{\partial\log X_f}{\partial\log W_f}d\log W_f + \sum_{f'\neq f}\frac{\partial\log X_f}{\partial\log W_{f'}}d\log W_{f'}, \\
&= -\left(\sum_{f'\neq f}\theta_{f'}\sigma_{ff'}\right)d\log W_f + \sum_{f'\neq f}\theta_{f'}\sigma_{ff'}d\log W_{f'},
\end{aligned}$$

where in the last equality we have used the expression for the own price elasticity from equation (4). Accordingly, we can write the above equation in vector form as

$$d \log \boldsymbol{X} = -\boldsymbol{D}\, d \log \boldsymbol{W},$$

where we have organized the vectors as $\boldsymbol{X} \equiv (X_\ell, X_h, \cdots, X_e)'$ and $\boldsymbol{W} \equiv (W_\ell, W_h, \cdots, W_e)'$, and where we have defined the matrix $\boldsymbol{D}$ according to equation (A.29). Note that matrix $\boldsymbol{D}$ is not invertible since factor demand is homogeneous of degree zero and we have $\boldsymbol{D}\mathbf{1} = \mathbf{0}$. Now, define the vectors $\widetilde{\boldsymbol{X}} \equiv \left(X_f / X_\ell, \cdots\right)'$ and $\widetilde{\boldsymbol{W}} \equiv \left(W_f / W_\ell, \cdots\right)'$ as the vectors of the demand and prices for factors $f \in \mathcal{F} \backslash \{\ell, e\}$. Using the definitions in equation (A.30) and the additional vector $\widetilde{\mathbf{1}}_e \equiv (0, \cdots, 0, 1)'$, we can now write

$$\begin{aligned}
d \log \widetilde{\boldsymbol{X}} &= \widetilde{\boldsymbol{P}}_R'\, d \log \boldsymbol{X}, \\
&= -\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{D}\, d \log \boldsymbol{W}, \\
&= -\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{D}\, \left(d \log \boldsymbol{W} - \mathbf{1} d \log W_\ell\right), \\
&= -\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{D}\, \widetilde{\boldsymbol{P}}_L\, d \log \widetilde{\boldsymbol{W}} - \widetilde{\boldsymbol{P}}_R'\, \boldsymbol{D}\, \widetilde{\mathbf{1}}_e\, d \log \left(\frac{W_e}{W_\ell}\right).
\end{aligned} \tag{A.35}$$

Since factor supplies are not impacted by the change in the price of equipment $\frac{d \log\left(\overline{X}_f / \overline{X}_\ell\right)}{d \log W_e} = 0$ for each $f \notin \{\ell, e\}$, the left hand side of equation (A.35) is zero. We thus obtain the following equation

$$\begin{aligned}
d \log \widetilde{\boldsymbol{W}} &= -\left(\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{D}\, \widetilde{\boldsymbol{P}}_L\right)^{-1} \left(\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{D}\, \widetilde{\mathbf{1}}_e\right) d \log W_e, \\
&= -\left(\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{D}\, \widetilde{\boldsymbol{P}}_L\right)^{-1} \left(D_{fe} - D_{\ell e}\right)_{f \in \mathcal{F} \backslash \{\ell, e\}} d \log W_e, \\
&= -\left(\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{D}\, \widetilde{\boldsymbol{P}}_L\right)^{-1} \theta_e \left(\sigma_{\ell e} - \widetilde{\boldsymbol{\sigma}}_e\right) d \log W_e,
\end{aligned}$$

Since $0 = \frac{d \log\left(\overline{X}_f / \overline{X}_\ell\right)}{d \log W_e}$, we find for each $f \notin \{\ell, e\}$

$$\begin{aligned}
0 &= \frac{d \log\left(X_f / X_\ell\right)}{d \log W_e}, \\
&= \frac{\partial \log X_f / X_\ell}{\partial \log W_e} + \frac{\partial \log X_f / X_\ell}{\partial \log W_f} \frac{d \log W_f}{d \log W_e} + \sum_{f' \notin \{e, f, \ell\}} \frac{\partial \log X_f / X_\ell}{\partial \log W_{f'}} \frac{d \log W_{f'}}{d \log W_e}, \\
&= -\theta_e \left(\sigma_{\ell e} - \sigma_{fe}\right) - \left(\sum_{f' \neq f} \theta_{f'} \sigma_{f f'} + \theta_f \sigma_{\ell f}\right) \frac{d \log W_f}{d \log W_e}
\end{aligned}$$

$$+\sum_{f'\notin\{e,f,\ell\}}\theta_{f'}\left(\sigma_{ff'}-\sigma_{\ell f'}\right)\frac{d\log W_{f'}}{d\log W_e}.$$

Rewriting the above expression in matrix form, we find

$$-\theta_e\left(\sigma_{\ell e}-\boldsymbol{\sigma}_{\cdot e}\right)=\left[\mathrm{diag}\left(\boldsymbol{\Sigma\theta}\right)+\mathbf{1}\boldsymbol{\sigma}'_{\cdot\ell}\mathrm{diag}\left(\boldsymbol{\theta}\right)-\boldsymbol{\Sigma}\mathrm{diag}(\boldsymbol{\theta})\right]\frac{d\log\boldsymbol{W}}{d\log W_e}\tag{A.36}$$

which leads to the desired result presented in equation (A.28). □

Proof for Proposition A.3. We follow the same steps as in the proof of Proposition 2 above. First, we have the following relationship between factor demand and firm-specific factor price shocks

$$\begin{aligned}d\log X_f&=\sum_i\frac{X_{fi}}{X_f}\left[\frac{\partial\log\left(X_{fi}/Y_i\right)}{\partial\log W_{f'i}}d\log W_{f'i}+d\log Y_i\right],\\&=\sum_i\frac{X_{fi}}{X_f}\left[\frac{\partial\log\left(X_{fi}/Y_i\right)}{\partial\log W_{f'i}}\omega_{f'i}\,d\log\overline{W}_{f'}+d\log Y_i\right].\end{aligned}\tag{A.37}$$

Moreover, the firm's share of aggregate factor demand now satisfies

$$\frac{X_{fi}}{X_f}=\frac{Y_iC_i}{YC}\frac{W_{fi}X_{fi}/\left(Y_iC_i\right)}{\overline{W}_fX_f/\left(YC\right)}\frac{\overline{W}_f}{W_{fi}}=\Lambda_i\widehat{\theta}_{fi}\frac{\overline{W}_f}{W_{fi}}.\tag{A.38}$$

Finally, we can rewrite equation (A.33) as

$$\begin{aligned}d\log Y_i&=-\varepsilon\left(\frac{\partial\log C_i}{\partial\log W_{f'i}}d\log W_{f'i}-d\log C_s\right)-\eta\left(d\log C_s-d\log C\right),\\&=-\varepsilon\left(\frac{\partial\log C_i}{\partial\log W_{f'i}}d\log W_{f'i}-\sum_i\Lambda_{i|s}\theta_{f'i}d\log W_{f'i}\right)\\&\qquad-\eta\left(\sum_i\Lambda_{i|s}\theta_{f'i}d\log W_{f'i}-\sum_i\Lambda_i\theta_{f'i}d\log W_{f'i}\right),\\&=-\varepsilon\left(\theta_{f'i}\omega_{f'i}-\sum_i\Lambda_{i|s}\theta_{f'i}\omega_{f'i}\right)d\log\overline{W}_{f'}-\eta\left(\sum_i\Lambda_{i|s}\theta_{f'i}\omega_{f'i}-\sum_i\Lambda_i\theta_{f'i}\omega_{f'i}\right)d\log\overline{W}_{f'},\\&=-\varepsilon\left(\theta_{f'i}\omega_{f'i}-\mathbb{E}_{i|s}\left[\theta_{f'i}\omega_{f'i}\right]\right)d\log\overline{W}_{f'}-\eta\left(\theta_{f's}\omega_{f's}-\mathbb{E}_s\left[\theta_{f's}\omega_{f's}\right]\right)d\log\overline{W}_{f'},\end{aligned}\tag{A.39}$$

where in the second equality we have used Shephard's lemma for the unit cost of sectoral production, and in the fourth equality we have let $\theta_{f's} \equiv \sum_i \Lambda_{i|s} \theta_{f'i}$ and defined $\omega_{f's}$ as in equation (A.14).

Substituting equations (A.39) and (A.38) in equation (A.37), and using the definition (A.16), we find

$$\begin{aligned}
\sigma^{\omega}_{ff'} &= \sum_i \Lambda_i \frac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi} \widehat{\theta}_{f'i} \omega_{f'i} \sigma_{ff',i} - \varepsilon \sum_i \Lambda_i \frac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi} \left(\widehat{\theta}_{f'i} \omega_{f'i} - \mathbb{E}_{i|s} \left[\widehat{\theta}_{f'i} \omega_{f'i} \right] \right) \\
&\qquad - \eta \sum_i \Lambda_i \frac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi} \left(\widehat{\theta}_{f's} \omega_{f's} - \mathbb{E}_s \left[\widehat{\theta}_{f's} \omega_{f's} \right] \right), \\
&= \sum_i \Lambda_i \frac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi} \widehat{\theta}_{f'i} \omega_{f'i} \sigma_{ff',i} - \varepsilon \sum_s \Lambda_s \sum_{i \in \mathcal{I}_s} \Lambda_{i|s} \frac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi} \left(\widehat{\theta}_{f'i} \omega_{f'i} - \mathbb{E}_{i|s} \left[\widehat{\theta}_{f'i} \omega_{f'i} \right] \right) \\
&\qquad - \eta \sum_s \Lambda_s \sum_{i \in \mathcal{I}_s} \Lambda_{i|s} \frac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi} \left(\widehat{\theta}_{f's} \omega_{f's} - \mathbb{E}_s \left[\widehat{\theta}_{f's} \omega_{f's} \right] \right), \\
&= \mathbb{E}_i \left[\tfrac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi} \widehat{\theta}_{f'i} \omega_{f'i} \sigma_{ff',i} \right] - \varepsilon \, \mathbb{E}_s \left[\mathbb{C}_{i|s} \left(\tfrac{\overline{W}_f}{W_{fi}} \widehat{\theta}_{fi}, \widehat{\theta}_{f'i} \omega_{f'i} \right) \right] - \eta \, \mathbb{C}_s \left(\tfrac{\overline{W}_f}{W_{fs}} \widehat{\theta}_{fs}, \widehat{\theta}_{f's} \omega_{f's} \right).
\end{aligned}$$

To compute the elasticity for factor payments $\sigma^{*,\omega}_{ff'}$, we follow similar steps

$$\begin{aligned}
\sigma^{*,\omega}_{ff'} &\equiv 1 + \frac{1}{\theta_{f'}} \frac{\partial \log \theta_f}{\partial \log \overline{W}_{f'}}, \\
&= \frac{1}{\theta_{f'}} \frac{\partial \log \left(\sum_i W_{fi} X_{fi} \right)}{\partial \log \overline{W}_{f'}}, \\
&= \frac{1}{\theta_{f'}} \sum_i \frac{W_{fi} X_{fi}}{\sum_{i'} W_{fi'} X_{fi'}} \left[\frac{\partial \log \left(X_{fi} / Y_i \right)}{\partial \log \overline{W}_{f'}} + \frac{\partial \log Y_i}{\partial \log \overline{W}_{f'}} \right], \\
&= \frac{1}{\theta_{f'}} \sum_i \Lambda_i \widehat{\theta}_{fi} \left[\frac{\partial \log \left(X_{fi} / Y_i \right)}{\partial \log \overline{W}_{f'}} + \frac{\partial \log Y_i}{\partial \log \overline{W}_{f'}} \right], \\
&= \frac{1}{\theta_{f'}} \sum_i \Lambda_i \widehat{\theta}_{fi} \left[\theta_{f'i} \omega_{f'i} \sigma_{ff',i} + \frac{\partial \log Y_i}{\partial \log \overline{W}_{f'}} \right].
\end{aligned}$$

Once again, substituting equation (A.39) in the above equation and following the same steps as before leads to the desired result. □

Proof for Proposition A.4. Applying the same logic as the previous proofs, since $\frac{d \ln \overline{X}_h / \overline{X}_\ell}{d \ln \overline{W}_e} =$

0, we have

$$
\begin{aligned}
0 &= \frac{\partial \ln X_h/X_\ell}{\partial \ln \overline{W}_e} + \left(\frac{\partial \ln X_h/X_\ell}{\partial \ln W_h}\right) \frac{\partial \ln W_h}{\partial \ln \overline{W}_e}, \\
&= -\theta_e \left(\sigma^\omega_{\ell e} - \sigma^\omega_{he}\right) - \left(\theta_\ell \sigma_{h\ell} + \theta_e \sigma_{he} + \theta_h \sigma_{h\ell}\right) \frac{\partial \ln W_h}{\partial \ln \overline{W}_e},
\end{aligned}
$$

leading to equation (A.22).

The same argument as that in the proof of Proposition 1 still applies here. Since relative skill demand X_h/X_ℓ does not change, we have

$$
\begin{aligned}
\frac{d \log \Psi}{d \log \overline{W}_e} &= \frac{d}{d \log \overline{W}_e} \log \left(\frac{\overline{W}_h}{\overline{W}_\ell}\right), \\
&= \frac{d}{d \log \overline{W}_e} \log \left(\frac{\overline{W}_h X_h}{\overline{W}_\ell X_\ell}\right), \\
&= \frac{d}{d \log \overline{W}_e} \log \left(\frac{\theta_h}{\theta_\ell}\right), \\
&= \frac{\partial \log \theta_h/\theta_\ell}{\partial \log \overline{W}_e} + \left(\frac{\partial \log \theta_h/\theta_\ell}{\partial \log W_h}\right) \frac{d \log W_h}{d \log \overline{W}_e}, \\
&= -\theta_e \left(\sigma^{*,\omega}_{\ell e} - \sigma^{*,\omega}_{he}\right) + \left[\frac{\partial \log \theta_h}{\partial \log W_h} - \theta_h \left(\sigma^*_{\ell h} - 1\right)\right] \frac{d \log W_h}{d \log \overline{W}_e}. \qquad \text{(A.40)}
\end{aligned}
$$

Note that we have

$$
\begin{aligned}
\frac{\partial \log \theta_f}{\partial \log W_f} &= \frac{1}{\theta_f} \frac{\partial}{\partial \log W_f} \left(1 - \sum_{f' \neq f} \theta_{f'}\right), \\
&= -\frac{1}{\theta_f} \sum_{f' \neq f} \theta_{f'} \frac{\partial \log \theta_{f'}}{\partial \log W_f}, \\
&= -\frac{1}{\theta_f} \sum_{f' \neq f} \theta_{f'} \theta_f \left(\sigma^*_{f'f} - 1\right), \\
&= -\sum_{f' \neq f} \theta_{f'} \sigma^*_{f'f} + 1 - \theta_f.
\end{aligned}
$$

Substituting this expression in equation (A.40), we find

$$
\begin{aligned}
\frac{d \log \Psi}{d \log \overline{W}_e} &= -\theta_e \left(\sigma^{*,\omega}_{\ell e} - \sigma^{*,\omega}_{he}\right) + \left[-\theta_\ell \sigma^*_{\ell h} - \theta_e \sigma^*_{eh} + 1 - \theta_h - \theta_h \left(\sigma^*_{\ell h} - 1\right)\right] \frac{d \log W_h}{d \log \overline{W}_e}, \\
&= \frac{d \log W_h}{d \log \overline{W}_e} - \theta_e \left(\sigma^{*,\omega}_{\ell e} - \sigma^{*,\omega}_{he}\right) - \left[\left(1 - \theta_e\right) \sigma^*_{\ell h} + \theta_e \sigma^*_{eh}\right] \frac{d \log W_h}{d \log \overline{W}_e},
\end{aligned}
$$

$$= \frac{d\log W_h}{d\log W_e} - \left[(1-\theta_e)\,\sigma^*_{\ell h} + \theta_e \sigma^*_{eh}\right]\left(\frac{d\log W_h}{d\log \overline{W}_e} + \frac{\theta_e\left(\sigma^*_{\ell e} - \sigma^*_{he}\right)}{(1-\theta_e)\,\sigma^*_{\ell h} + \theta_e \sigma^*_{he}}\right),$$

leading to equation (A.23). □

Proof for equation (A.25) and Proposition A.5. We first establish equation (A.25). Since $\theta_c = \sum_{f\in\mathcal{F}_c}\theta_f$, we have

$$\frac{1}{\theta_{f'}}\frac{\partial\log\theta_c}{\partial\log W_{f'}} = \frac{1}{\theta_{f'}\theta_c}\sum_{f\in\mathcal{F}_c}\frac{\partial\theta_f}{\partial\log W_{f'}} = \frac{1}{\theta_c}\sum_{f\in\mathcal{F}_c}\theta_f\left(\sigma^*_{ff'}-1\right) = \sum_{f\in\mathcal{F}_c}\frac{\theta_f}{\theta_c}\sigma^*_{ff'} - 1,$$

where the second equality uses $\partial\log\theta_f/\partial\log W_{f'} = \theta_{f'}(\sigma^*_{ff'}-1)$, which follows from the definition of $\sigma^*_{ff'}$ in equation (A.15) for $f \neq f'$ (and which applies for all $f\in\mathcal{F}_c$ since $f'\notin\mathcal{F}_c$). Adding 1 to both sides yields the second equality in equation (A.25).

We next establish Proposition A.5. The case of $f'\notin\mathcal{F}_c$ follows immediately from the definition of $\sigma^*_{cf'}$ in equation (A.25). In the case of $f'\in\mathcal{F}_c$, we split the sum over $\mathcal{F}_c$ into the own-price term $f = f'$ and the cross-price terms $f\in\mathcal{F}_c\setminus\{f'\}$:

$$\begin{aligned}
\frac{\partial\log\theta_c}{\partial\log W_{f'}} &= \sum_{f\in\mathcal{F}_c}\frac{\theta_f}{\theta_c}\frac{\partial\log\theta_f}{\partial\log W_{f'}},\\
&= \frac{\theta_{f'}}{\theta_c}\frac{\partial\log\theta_{f'}}{\partial\log W_{f'}} + \sum_{f\in\mathcal{F}_c\setminus\{f'\}}\frac{\theta_f}{\theta_c}\theta_{f'}\left(\sigma^*_{ff'}-1\right),\\
&= -\frac{\theta_{f'}}{\theta_c}\left(1-\theta_{f'}\right)\left(\sigma^*_{\overline{f}'f'}-1\right) + \theta_{f'}\frac{\theta_{\overline{c}_{f'}}}{\theta_c}\left(\sigma^*_{\overline{c}_{f'}f'}-1\right),\\
&= \theta_{f'}\left[\frac{\theta_{\overline{c}_{f'}}}{\theta_c}\left(\sigma^*_{\overline{c}_{f'}f'}-1\right) - \frac{\theta_{\overline{f}'}}{\theta_c}\left(\sigma^*_{\overline{f}'f'}-1\right)\right],
\end{aligned}$$

where in the third equality we have applied equation (A.25) to the composite $\overline{c}_{f'}$ to obtain $\sum_{f\in\mathcal{F}_c\setminus\{f'\}}\theta_f(\sigma^*_{ff'}-1) = \theta_{\overline{c}_{f'}}(\sigma^*_{\overline{c}_{f'}f'}-1)$, and to the composite $\overline{f}'$ to write the own-price elasticity as

$$\frac{\partial\log\theta_{f'}}{\partial\log W_{f'}} = -(1-\theta_{f'})(\sigma^*_{\overline{f}'f'}-1),$$

which itself follows from the adding-up identity $\sum_{f\in\mathcal{F}}\partial\theta_f/\partial\log W_{f'} = 0$ together with $\partial\log\theta_f/\partial\log W_{f'} = \theta_{f'}(\sigma^*_{ff'}-1)$ for $f\neq f'$; and where the last equality uses $\theta_{\overline{f}'} = 1-\theta_{f'}$. This yields the second case of equation (A.26).

Application to the labor composite. As an example, consider the three-factor model with $\mathcal{F} = \{\ell, h, e\}$ and the composite labor factor $n \equiv \ell + h$ (so $\mathcal{F}_n = \{\ell, h\}$). For $f' = h \in \mathcal{F}_n$, the second case of equation (A.26) applies with $\overline{c}_{f'} = \overline{n}_h = \{\ell\}$ (so $\theta_{\overline{n}_h} = \theta_\ell$ and

$\sigma^*_{\overline{n}_h h} = \sigma^*_{\ell h}$) and $\overline{f}' = \overline{h} = \{\ell, e\}$ (so $\theta_{\overline{h}} = \theta_\ell + \theta_e$ and, using equation (A.25), $\sigma^*_{\overline{h}h} = (\theta_\ell \sigma^*_{\ell h} + \theta_e \sigma^*_{eh})/(\theta_\ell + \theta_e)$). Substituting,

$$\begin{aligned}\frac{\partial \log \theta_n}{\partial \log W_h} &= \theta_h \left[\frac{\theta_\ell}{\theta_n}\left(\sigma^*_{\ell h} - 1\right) - \frac{\theta_\ell + \theta_e}{\theta_n}\left(\sigma^*_{\overline{h}h} - 1\right)\right] \\ &= \frac{\theta_h}{\theta_n}\left[\theta_\ell\left(\sigma^*_{\ell h} - 1\right) - \theta_\ell\left(\sigma^*_{\ell h} - 1\right) - \theta_e\left(\sigma^*_{eh} - 1\right)\right] \\ &= \frac{\theta_e \theta_h}{\theta_n}\left(1 - \sigma^*_{eh}\right),\end{aligned}$$

where the second equality uses the expansion of $\sigma^*_{\overline{h}h}$ above. For $f' = e \notin \mathcal{F}_n$, the first case of equation (A.26) gives $\partial \log \theta_n / \partial \log W_e = \theta_e(\sigma^*_{ne} - 1)$ directly. □

A.5 Aggregation in the Data

In this section, we present the second-order approximations that we use in our empirical exercise to perform the aggregation results presented in Section 5.

We first define the following notation, using the relative-intensity notation in equation (14). Consider a function $v(\cdot)$ of time-varying factor intensities $\boldsymbol{\theta}_t \equiv \left(\theta_{fi,t}, \theta_{f,t}\right)_{i,f}$, elasticities $\boldsymbol{\sigma}_t \equiv \left(\sigma_{ff'i,t}, \sigma\right)$, and shock exposure weights $\boldsymbol{\omega}_t \equiv \left(\omega_{f'i,t}\right)$ defined following $\omega_{f'i,t} \equiv \Delta \log W_{f'i,t} / \Delta \log \overline{W}_{f't}$ with $\Delta \log \overline{W}_{f't} \equiv \sum_i \Lambda_i \widehat{\theta}_{f'i} \Delta \log W_{f'i}$ at time t. Define the time averaging operator $\langle \cdot \rangle$ for the function between two consecutive periods $t-1$ and t as

$$\langle v(\boldsymbol{\theta}_t, \boldsymbol{\sigma}_t, \boldsymbol{\omega}_t) \rangle \equiv \frac{1}{2}\left(v(\boldsymbol{\theta}_t, \boldsymbol{\sigma}_t, \boldsymbol{\omega}_t) + v(\boldsymbol{\theta}_{t-1}, \boldsymbol{\sigma}_{t-1}, \boldsymbol{\omega}_t)\right),$$

where we keep the same fixed exposure weights $\boldsymbol{\omega}_t$ in both terms.

We now apply a second order approximation of the relative changes in the skill supply and demand in the general case with heterogeneous changes in equipment prices, following the proof of Proposition A.4 in Appendix A.4.1 above, to find

$$\begin{aligned}\Delta \log\left(\frac{\overline{X}_{h,t}}{\overline{X}_{\ell,t}}\right) &= \Delta \log\left(\frac{X_{h,t}}{X_{\ell,t}}\right), \\ &\approx -\left\langle \theta_{e,t}\left(\sigma^\omega_{\ell e,t} - \sigma^\omega_{he,t}\right)\right\rangle \Delta \log \overline{W}_{e,t} - \left\langle \left(1 - \theta_{e,t}\right)\sigma_{h\ell,t} + \theta_{e,t}\sigma_{he,t}\right\rangle \Delta \log W_{h,t}.\end{aligned} \tag{A.41}$$

This leads to the following second-order approximation of the predicted change in the skill premium:

$$\Delta \log W_{h,t} \approx -\frac{\left\langle \theta_{e,t}\left(\sigma^{\omega}_{\ell e,t}-\sigma^{\omega}_{he,t}\right)\right\rangle}{\left\langle \left(1-\theta_{e,t}\right)\sigma_{h\ell,t}+\theta_{e,t}\sigma_{he,t}\right\rangle}\Delta \log \overline{W}_{e,t} - \frac{1}{\left\langle \left(1-\theta_{e,t}\right)\sigma_{h\ell,t}+\theta_{e,t}\sigma_{he,t}\right\rangle}\Delta \log\left(\frac{\overline{X}_{h,t}}{\overline{X}_{\ell,t}}\right). \tag{A.42}$$

In practice, we find that the observed shifts in the skill premium $\Delta \log \widehat{W}_{h,t}$ do not equalize with the left hand side of equation (A.42). To close this gap, consider uniform skill-biased shifts in technology of the form

$$\Delta a_t \equiv \log\left(\frac{Z_{hi,t}/Z_{\ell i,t}}{Z_{hi,t-1}/Z_{\ell i,t-1}}\right).$$

Then, we can further generalize equation (A.41) to write

$$\begin{aligned}\Delta \log\left(\frac{\overline{X}_{h,t}}{\overline{X}_{\ell,t}}\right) &= \Delta \log\left(\frac{X_{h,t}}{X_{\ell,t}}\right),\\ &\approx -\left\langle \theta_{e,t}\left(\sigma^{\omega}_{\ell e,t}-\sigma^{\omega}_{he,t}\right)\right\rangle \Delta \log \overline{W}_{e,t} - \left\langle \left(1-\theta_{e,t}\right)\sigma_{h\ell,t}+\theta_{e,t}\sigma_{he,t}\right\rangle \left(\Delta \log W_{h,t} - \Delta a_t\right).\end{aligned}$$

This allows us to infer equivalent changes

$$\Delta a_t \approx \Delta \log \widehat{W}_{h,t} + \frac{\left\langle \theta_{e,t}\left(\sigma^{\omega}_{\ell e,t}-\sigma^{\omega}_{he,t}\right)\right\rangle}{\left\langle \left(1-\theta_{e,t}\right)\sigma_{h\ell,t}+\theta_{e,t}\sigma_{he,t}\right\rangle}\Delta \log \overline{W}_{e,t} + \frac{1}{\left\langle \left(1-\theta_{e,t}\right)\sigma_{h\ell,t}+\theta_{e,t}\sigma_{he,t}\right\rangle}\Delta \log\left(\frac{\overline{X}_{h,t}}{\overline{X}_{\ell,t}}\right), \tag{A.43}$$

Note that in general we can interpret the inferred shifts in equation (A.43) either as changes in production technology or equivalent changes in distortions that effectively act as taxes on the hiring of unskilled workers for firms.

A.6 Microfoundations for the Factor Price Wedges

A.6.1 Heterogeneous Equipment

We provide a microfoundation for firm-specific equipment prices stemming from variation in the composition of equipment capital across firms. We make two key assumptions. First, we assume that the stock of equipment for firm i is a CRS aggregate across J varieties of equipment capital given by

$$X_{ei} = \mathcal{G}_i\left(X_{ei1}, \cdots, X_{eij}, \cdots, X_{eiJ}\right),$$

where each variety j is a combination of equipment type and origin country (from which the variety is purchased). Second, we assume that the aggregator $\mathcal{G}_i$ is such that the marginal product of all varieties is bounded above by a constant

$$\frac{\partial G_i}{\partial X_{eij}} \leq B < \infty, \qquad \forall i, j,$$

ensuring that equipment variety demand of all firms has a finite choke price above which demand falls to zero.

We introduce firm×variety-specific price distortions $W_{eij} = T_{eij} W_{ej}$, which may be driven, for instance, by firm-specific iceberg costs of sourcing a given variety from the corresponding origin country. Given these assumptions, the price of equipment capital for firm i is given by

$$W_{ei} = \mathcal{P}_i \left(T_{ei1} W_{e1}, \cdots, T_{eij} W_{ej}, \cdots, T_{eiJ} W_{eJ} \right),$$

where $\mathcal{P}_i$ is the unit cost function corresponding to the CRS aggregator $\mathcal{G}_i$.

The assumption of the choke price implies that for a given collection of firm-specific wedges $\left(T_{eij}\right)_{j=1}^{J}$, the firm may find its firm-specific price W_{eij} to be above the variety-specific choke price for some varieties, and therefore the extensive margin of the composition of equipment varieties is heterogeneous across firms. When we map our framework to the data on firm-level imports, this is indeed what we find, since the composition and the sources of equipment goods imported vary across firms. However, since marginal equipment varieties have a zero marginal product, changes in the extensive margin of sourcing decisions due to small firm-specific price changes do not affect the firm-level equipment price W_{ei} to the first order of approximation.

A.6.2 Compensating Differentials

Here, we provide a microfoundation for firm- and skill-type-specific wages stemming from variations in compensating differentials across firms. In this microfoundation, the labor market is perfectly competitive.

Suppose that firms are characterized by variations in amenities that are specific to worker skill. More specifically, if a high-skilled (or low-skilled) worker is employed by firm i, then her utility is her real income divided by T_{hi} (or $T_{\ell i}$). The market equilibrium then leads to firm-specific compensating differentials that are equivalent to amenity differences.

For two firms i and i' to have positive employment of skilled (or unskilled) labor in a competitive labor market, their wages must satisfy $W_{hi}/T_{hi} = W_{hi'}/T_{hi'}$ (or $W_{\ell i}/T_{\ell i} = W_{\ell i'}/T_{\ell i'}$). Letting W_h and W_ℓ denote the shadow wages of skilled and unskilled labor, which we set equal to the wage of a (potentially counterfactual) firm i' with $T_{hi'} = 1$ or $T_{\ell i'} = 1$, we then have $W_{hi} = W_h T_{hi}$ and $W_{\ell i} = W_\ell T_{\ell i}$ for all i, exactly as in our model of Section A.3.2.

B Data Appendix

In this appendix, we present a comprehensive description of the data sources and the main variables used in the analysis. Additionally, we discuss how we construct the sample used in the estimation of the micro-elasticities and the sample used in the aggregation exercise.

B.1 List of All Data Sources Used

BRN: The *Bénéfices Réels Normaux* dataset provides information on income and balance sheets for companies with sales exceeding certain thresholds as well as some smaller firms that opt in.[A2] The data are sourced from the official document CERFA 2050-9, which serves as the French counterpart to the U.S. IRS Form 1120 (Corporate Income Tax Return). In 2003, the dataset covered 24% of French firms, accounting for over 94.3% of total gross output (INSEE, 2004) and over 90% of the aggregate value of trade flows in customs records (authors' computation). The available data span from 1993 to 2009 and cover employment, sales, value added, wage bill, payroll taxes, and a breakdown of investment. As we discuss in detail in Sections B.2 and C.1, we use variables retrieved from this dataset to compute firm-level measures of gross output, intermediates, as well as the stock and user cost of capital equipment.

DADS: The *Déclarations Annuelles de Données Sociales* (INSEE, 2010) is a matched employer-employee dataset that covers the entire population of French private sector workers. Among the different versions of the dataset made available by INSEE, we exploit DADS Poste (*Fichiers Régionaux des Postes*), which provides information at the level of each job spell. The information includes the worker's establishment, wage, hours worked, and 2-digit occupational classification. As we will discuss in detail in Section C.2 below, firm-level employment and wage bill data are computed by aggregating worker-level data on

[A2] In 2007, the threshold was €763,000 for trade and real estate sector firms and €230,000 for firms operating in all other sectors INSEE (2004).

hours worked and wages.

Customs dataset: Firm-level international transaction data are provided by the *French Directorate-General of Customs and Indirect Taxes (DGDDI)*, offering information on the annual value of imports and exports by country of origin/destination and CN8 product for all firms involved in international transactions. This dataset combines information gathered under two separate legal frameworks: the *Déclaration d'Echange de Biens (DEB)*, which addresses intra-EU trade flows, and the *Document Administratif Unique (DAU)*, which pertains to transactions with non-EU countries. The number of variables collected within each statistical framework depends on thresholds specific to the firm and the transactions (Bergounhon et al., 2018). For intra-EU DEB transactions, the information available depends on the firm's size, measured by the total value of intra-EU trade it conducts during the relevant calendar year. During the study period, the minimum threshold requiring firms to provide the information used in this study increased from €38,000 (1993-2000) to €100,000 (2002-2006), eventually reaching €150,000 in 2007. For the purpose of this study, we apply the latter threshold throughout the entire sample to ensure consistency. On the other hand, the extra-EU DAU data include all transactions above €1,000 or 1,000 kilos. Data available from 1994 to 2007 include the value, weight, and number of units exported by each firm. These data are used to compute firm-level equipment variables (see Section C.1.1), as well as the trade instruments presented in Section 4.2.

Répertoire Sirene: The *Répertoire Sirene* is a national directory of companies and their establishments (business register), managed by the National Institute of Statistics and Economic Studies (INSEE, 2021).

BACI Dataset: The *Base pour l'Analyse du Commerce International* dataset (Gaulier and Zignago, 2010) contains information on bilateral trade flows by year and product. We use this dataset to measure the world-export-supply of intermediates, which is a key component of the world-export-supply instrument presented in Section 4.2.

KLEMS: Data on depreciation rates by industry and asset category, as well as industry-level gross output, are retrieved from the *Growth and Productivity Accounts* published by EU-KLEMS (*EU level analysis of capital (K), labor (L), energy (E), materials (M), and service (S) inputs*). The dataset includes industry-level measures of economic growth, productivity, employment and capital formation for all European Union member states from 1970 onwards (Stehrer et al., 2019). We use these data to compute equipment stock and user cost of capital, as discussed in Section C.1.4 and C.1.5, and the skill-specific wage instrument presented in Section 4.2.

Eurostat: We retrieve industry-level aggregate data from the *Eurostat National Accounts – Main Aggregates* series (Eurostat, 2019). This dataset, compiled in accordance with the European System of Accounts (ESA, 2013), includes a coherent and consistent set of macroeconomic indicators, which provide an overall picture of the economic situation of each country. For the purpose of this study, we exploit information on aggregate levels and price indices of capital and investment by asset type and by sector, as well as industry-level data on value added and employment. These data are used in the construction of industry-specific prices and depreciation indices (see Section B.2).

IMF WEO: Data on exchange rates are retrieved from the *IMF World Economic Outlook Database*. These series are used to construct the exchange rate instrument presented in Section 4.2.

WIOD: We use the Socio-Economic Accounts of the *World Input-Output Database* (2013 Release) to construct an alternative proxy for low-skilled wages used as the numeraire in the construction of the aggregate shock to equipment prices (see Section C.1.7).

FRED: Data on historical government bond yields are retrieved from the *Federal Reserve Economic Data (FRED)*.

INSEE: Data on the monthly rate of the nominal hourly minimum wage – *Salaire Minimum Interprofessionnel de Croissance (Smic)* – are provided by the *Institut National de la Statistique et des Études Économiques (INSEE)*.

B.2 Variable Definition

Gross Output: The gross output variable is based on data retrieved from the BRN dataset. It is defined as the sum of three components: sales (*Chiffre d'affaires nets, r310*), capitalised production (*Production immobilisée, r312*), and change in inventories (*Production stockée, r311*). The variable is reported as missing when its main component, sales, is missing or equal to zero.

Intermediates: The intermediates variable is also based on data retrieved from the

BRN dataset. It is defined as the sum of three components: purchases of goods and raw materials (*Achats de marchandises, matiéres premiéres et approvisionnements, r210+r212*), change in inventory of goods and raw materials (*Variation de stock des marchandises, matiéres premiéres et approvisionnements, r211 + r213*), and other supplies and external expenses (*Autres achats et charges externes, r214*). The variable is reported as missing when its main component, purchase of raw materials, is missing or equal to zero.

Value Added: We define value added as the difference between gross output and intermediates. This variable largely coincides with the definition of *valeur ajoutée hors taxe* (value added before taxes) provided by *Systéme Unifié des Statistiques d'Entreprises* (SUSE). The only difference is that we exclude two components: "other products" (*Autres produits, r315*) and "other charges" (*Autres charges, r222*). Despite this minor difference, the two variables have a correlation of 0.992.

Number of Employees: This variable is defined as the full-time equivalent (FTE) employment. We obtain this variable in two steps. First, we extract data on the number of hours worked reported for each employee of a firm in the DADS Poste and divide it by 1,820, which is the expected number of hours worked in a year by a full-time worker with a 35-hour week contract. To avoid discontinuities caused by the reform of the workweek system that took place during the study period, we cap our proxy at 1.[A3] Subsequently, we aggregate this variable at the firm level.

Observed wages: Hourly wages are obtained using data on gross wages and paid hours retrieved from DADS. Gross wages correspond to all the sums received by the employee under their employment contract,[A4] while paid hours represent the cumulative total of all periods during which the employee remained connected to a firm, including overtime, periods of illness, and work-related accidents, except for unpaid leave periods.

Equipment stock (book value): Firm-level data available in BRN include a breakdown of tangible capital by asset type. Our measure of domestic equipment is the sum of two components: Machinery, equipment and tools (*AR - Installations techniques, matériel et outillage industriels*) and Other tangible fixed assets (*AT - Autres immobilisations corporelles*).

Equipment depreciation account: Similar to the book value variable, the depreciation

[A3]Between 1998 and 2002, France gradually introduced a 35-hour workweek system.

[A4]This salary is understood before any mandatory deductions and is calculated based on the CSG base.

account variable is the sum of two components recorded in the BRN dataset: Machinery, equipment and tools (*AS - Installations techniques, matériel et outillage industriels*) and Other tangible fixed assets (*AU - Autres immobilisations corporelles*).

Disposed assets: The BRN variable (*Charges exceptionnelles sur opérations en capital, HF*) records the net book value of the disposed assets.

Total imports/exports (values): Firm-level data on imports and exports by partner country and HS6 product are retrieved from the Customs dataset.

Intermediate imports (values): Firm-level data on intermediate imports by partner country and HS6 product are retrieved from the Customs dataset and are used as weights to compute the world-export-supply instrument presented in Section 4.2.

Imported equipment (values and quantities): Data on imported equipment are retrieved from the Customs dataset. We identify imported equipment as all transactions belonging to three BEC Rev. 4 groups (*4 - Capital goods (except transport equipment), and parts and accessories thereof*; *521 - Transport equipment, other, industrial*; and *51 - Transport equipment, passenger motor cars*). These data are used to construct the equipment variables used in the estimation of production function elasticities (see Section C.1).

Depreciation Rates: We retrieve depreciation rates from KLEMS. Each industry is assigned an equipment depreciation rate by aggregating four ESA 2010 asset categories (*N11321G - Computer hardware*; *N11322G - Telecom equipment*; *N1131G - Transports*; and *N110G - Other machinery equipment and weapons*) using the time-varying industry-level investments in each asset group as weights.

Domestic Equipment Prices: Domestic prices are retrieved from Eurostat. The aggregation procedure implemented to obtain time-varying industry-specific equipment prices is symmetrical to the one used to compute depreciation rates.

Interest Rates: The interest rate used to compute our proxy for the user cost of capital (see Section C.1.5) is the interest associated with the 10-year French bond, as recorded by the Federal Reserve Bank of St. Louis.

Table B.1: Summary Statistics

	Source	Aggregation Sample						Estimation Sample (CRESH)					
		Obs.(Nb)	Mean	Sd	p25	p50	p75	Obs.(Nb)	Mean	Sd	p25	p50	p75
Output	BRN	3,316,958	6,891,873	144,982,709	497,922	1,060,867	2,567,584	583,428	26,176,778	333,348,924	1,553,101	3,819,448	10,656,651
Value Added	BRN	3,316,958	1,870,707	36,716,379	194,813	381,259	859,239	583,428	6,111,708	80,083,047	391,322	960,874	2,540,168
Equipment Payments	BRN	3,316,958	811,013	29,703,561	25,154	57,437	153,133	583,428	3,505,019	66,507,722	69,811	208,278	756,892
High-Skill Wage Bill	DADS	3,316,958	442,424	6,781,781	35,203	79,819	196,316	583,428	1,532,367	15,203,926	94,843	239,582	652,154
Low-Skill Wage Bill	DADS	3,316,958	387,176	8,318,276	32,196	85,198	221,246	583,428	1,105,042	16,534,159	65,043	188,010	559,132
High-Skill Working Hours	DADS	3,316,958	20,784	335,060	2,028	4,375	10,179	583,428	67,885	761,788	4,860	11,369	30,164
Low-Skill Working Hours	DADS	3,316,958	35,554	739,264	3,223	8,314	21,401	583,428	96,696	1,431,891	6,048	17,623	52,422
High-Skill Hourly Wage	DADS	3,316,958	19	9	13	17	22	583,428	21	9	16	20	25
Low-Skill Hourly Wage	DADS	3,316,958	10	3	9	10	11	583,428	11	3	9	11	12
Exports - Total	Customs	3,316,958	773,795	34,975,421	0	0	0	583,428	3,986,732	77,528,423	0	16,091	507,458
Imports - Total	Customs	3,316,958	847,049	32,298,511	0	0	0	583,428	4,444,240	75,414,149	457	275,203	1,253,646
Imports - Equipment Products	Customs	3,316,958	222,760	14,597,340	0	0	0	583,428	1,166,352	34,099,334	0	0	44,194
Imports - Intermediate Products	Customs	3,316,958	481,363	23,094,765	0	0	0	583,428	2,522,595	53,716,456	0	34,792	537,246

Notes: Observations is the number of firm-year pairs in manufacturing and non-financial services over the 1998-2007 period. The units for all variables are euros except for those involving working hours. The aggregation sample includes all firms used in the quantification exercise of Section 5, which is conditional on firms with positive equipment capital stock and employing both low- and high-skill workers. The estimation sample is restricted to the set of firms used in the estimation of the firm-level CRESH production elasticities (Table 4), which further conditions on firms importing intermediate products (and having both high-skilled and low-skilled stayers).

B.3 Industry and Occupational Classifications

B.3.1 Constant Industry

The available data report the NACE Rev. 1.1 industry classification between 1994 and 2007. However, certain macroeconomic statistics (such as aggregate prices and depreciation rates) are available exclusively at the NACE Rev. 2 level. We address this discontinuity by assigning each firm in our dataset a constant NACE Rev. 2 industry code.

To achieve the above assignment, we first construct a correspondence table. We retrieve information on the industry classification recorded by all firms registered in the Répertoire Sirene (see Section B.1). Each firm is assigned a 5-digit NAF code, with the first 4 digits corresponding to the NACE classification. We first select all firms that are present both in 2007 and 2008 and identify the industry codes reported in those two years. Subsequently, we compute the relative frequency of each NACE 1.1-NACE 2 match and assign each NACE 1.1 code to the NACE 2 code that provides the best match. This process results in a many-to-one crosswalk table between the two classifications, which we use to assign a NACE Rev. 2 code to all observations.

As a final step, we identify the first industry code associated with each firm over the study period and assign it to the observations recorded in subsequent years. Using this method, we create an industrial classification dataset where each private sector firm recorded between 1994 and 2007 is assigned a unique NACE Rev. 2 industrial code. We then univocally match these 4-digit NACE Rev. 2 codes to the 38 aggregate industries of the Eurostat classification.

B.3.2 Skill Groups

The DADS dataset assigns to each job spell (*poste*) a unique occupational code, based on a two-digit classification called *catégories socioprofessionnelles* (see Table B.2). Following prior work (e.g., Caliendo et al., 2015; Carluccio et al., 2015), we assign workers to two skill groups based on this classification. High-skill workers are those employed as Business Owners and Executives, Managers and Higher-level Intellectual Occupations, and Intermediate Occupations. Low-skill workers are employed as Clerks and Blue-collar Workers. To account for a minor variation in the classification over the study period, we aggregate the categories 31 (Health professionals and lawyers), 34 (Professors, scientific professions), and 37 (Administrative and commercial managers of enterprises). In Table B.3, we report summary statistics on hourly wages by skill group and sector.

Table B.2: List of CS occupational categories

Classification	Catégories socioprofessionnelles (CS)	Employment
High-skilled	**2 - Business Owners and Executives**	**134,223**
	21 - Owners/Executives of craft enterprises	31,390
	22 - Owners/Executives of industrial or commercial enterprises with $<$ 10 employees	25,243
	23 - Owners/Executives of industrial or commercial enterprises with $\geq$ 10 employees	77,590
	3 - Managers and Higher-level Intellectual Occupations	**1,427,430**
	31 - Health professionals and lawyers	
	34 - Professors and scientific occupations	702,631
	37 - Administrative and commercial managers of enterprises	
	35 - Information, arts, and entertainment occupations	52,767
	38 - Engineers and technical managers of enterprises	672,032
	4 - Intermediate Occupations	**2,225,872**
	42 - Primary school teachers and related occupations	40,983
	43 - Intermediate health and social work occupations	163,907
	46 - Intermediate administrative and commercial occupations in enterprises	1,048,093
	47 - Technicians	616,317
	48 - Foremen and supervisors	356,572
Low-skilled	**5 - Clerks**	**2,259,567**
	52 - Civilian employees and service agents in the public sector	170,785
	53 - Security and surveillance workers	103,658
	54 - Administrative employees of enterprises	989,189
	55 - Retail and sales employees	705,407
	56 - Personal services workers	290,528
	6 - Blue-collar Workers	**4,220,396**
	62 - Skilled industrial workers	1,364,460
	63 - Skilled craft workers	683,681
	64 - Drivers	523,166
	65 - Skilled workers in handling, warehousing, and transport	376,621
	67 - Unskilled industrial workers	955,746
	68 - Unskilled craft workers	313,533
	69 - Agricultural workers	3,189
Total employment		**10,267,488**

Notes: The table reports the two-digit *catégories socioprofessionnelles* (CS) recorded in DADS Poste and the assignment of each category to the high- and low-skill groups used throughout the analysis. Employment is the average full-time-equivalent employment by CS category, computed as the average of the annual employment figures over the sample period. Categories 31, 34, and 37 are pooled (see Section B.3.2); the reported figure is their combined employment.

Table B.3: Observed Wages by Occupational Category

	Hourly Wages										
	Low-skilled					High-skilled					
	Mean	10p	Median	90p	sd	Mean	10p	Median	90p	sd	N firms
Manufacturing [C]	12.2	9.2	11.7	15.8	2.8	21.6	15.8	21.2	27.6	5.5	56,679
Electricity, gas, water, and waste [D-E]	12.2	9.8	12	14.9	2.2	21.7	15.8	20.5	28.7	6	2,058
Construction [F]	11.8	9.6	11.6	14	2	21.6	14.5	20.6	29.5	7.6	39,726
Non-Financial Market Services [G-N]	10.9	8.4	10.4	13.9	2.5	19.4	12.9	18	27.3	7.1	213,130
Non Market Services [O-U]	10.5	8.7	10.3	12.3	1.9	17.9	12.7	16.9	23.4	7.7	20,103
Tot	11.3	8.7	10.8	14.5	2.6	20.2	13.4	19.2	27.5	6.8	331,696
	Paid Hours										
Manufacturing [C]	1,589	1,301	1,632	1,816	229	1,669	1,416	1,699	1,899	227	56,679
Electricity, gas, water, and waste [D-E]	1,503	1,268	1,527	1,717	202	1,586	1,312	1,617	1,828	226	2,058
Construction [F]	1,483	1,189	1,504	1,754	240	1,606	1,252	1,635	1,944	302	39,726
Non-Financial Market Services [G-N]	1,269	652	1,297	1,784	395	1,486	813	1,562	1,893	380	213,130
Non Market Services [O-U]	1,342	989	1,375	1,651	281	1,374	960	1,416	1,727	316	20,103
Tot	1,385	807	1,456	1,789	365	1,542	1,037	1,614	1,896	344	331,696

Notes: The table reports summary statistics on observed hourly wages (top panel) and paid hours (bottom panel) by skill group and sector, computed from DADS Poste over the sample period. Hourly wages are gross wage bills divided by paid hours; see Section C.2 for the construction and sample restrictions. The final column reports the number of firms.

B.3.3 Occupations, Education, and Labor Market Adjustment.

Our occupation-based skill classification rests on two assumptions: (i) occupations map closely onto skill in France; and (ii) employment protection in France does not prevent firms from adjusting their skill mix. Both assumptions are supported by the evidence. First, occupations in France are stratified by education, to a degree comparable to the United States. Table B.4 reports the distribution of the highest diploma across one-digit *catégories socioprofessionnelles*. In 2008, 61% of managers and higher intellectual professionals held a tertiary degree,[A5] compared with only 5% of clerks and 1% of blue-collar workers; conversely, clerks and blue-collar workers held at most a secondary diploma, or no diploma, in 85% and 96% of cases, respectively. This sharp educational stratification supports our occupation-based measure of skill.

Second, French firms adjust employment through separations, not only through reduced hiring. Recent work by Duhautois and Petit (2015), using all establishments with at least fifty employees over 1999–2010, shows that, even for open-ended contracts, hiring and separation rates trace the kinked "hockey-stick" profiles that Davis et al. (2012) document for the United States, with separations rising almost one-for-one as establishments contract. Figure B.1 replicates this exercise in our 2002–2007 sample. The figure plots establishment-level hiring and separation rates against establishment-level employment growth. Consistent with the benchmark in which worker flows closely track job flows, separations rise on the left of zero growth, while hires rise on the right. In our data, separations are the dominant margin of adjustment at contracting establishments, for both skill groups. Thus, French firms do not appear unusually constrained in adjusting employment through separations or in reorganizing their workforce.

Table B.4: Occupation and Highest Diploma Obtained

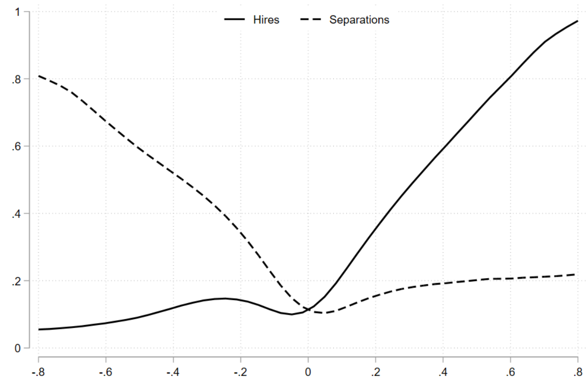
Classification	CS occupation	Highest diploma obtained			
		Higher education	Short higher / vocational	At most secondary	No diploma
High-skilled	Owners/Executives of enterprises with $\geq$ 10 empl.	23%	20%	43%	15%
	Managers and Higher-level Intellectual Occupations	61%	14%	23%	2%
	Intermediate Occupations	18%	30%	46%	5%
Low-skilled	Clerks	5%	10%	65%	20%
	Blue-collar Workers	1%	3%	62%	34%

Notes: Data from *Enquête emploi en continu 2008* (INSEE).

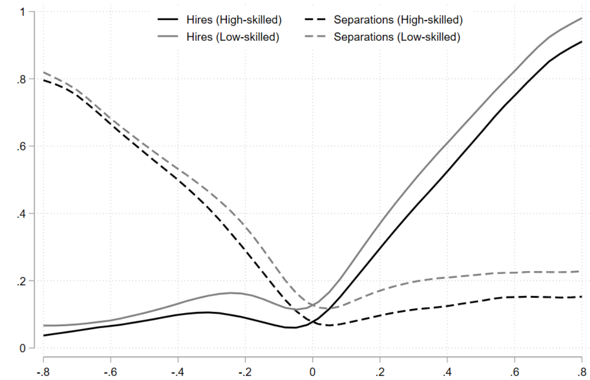
[A5] This figure is lower for owners and executives of enterprises with $\geq$ 10 employees, a subgroup that is considerably older than average (mean age 48, against 37 overall); reflecting the secular expansion of higher education, attainment is lower among these earlier birth cohorts. The subgroup represents 60% of the "Business Owners and Executives" category but only 2% of the broad high-skilled group.

Figure B.1: Worker flow rates as a function of establishment-level growth

(a) All workers



(b) High- vs. low-skilled



Notes: The figure plots establishment-level hiring and separation rates against the establishment's employment growth rate, following Davis et al. (2012) and Duhautois and Petit (2015). We use the DADS matched employer–employee files over 2002–2007. Because the establishment identifier at $t-1$ is unavailable before 2009, we cannot follow establishments across years and instead construct all measures *within* the calendar year. We first aggregate the underlying *poste* (job-spell) records to the worker–establishment level, recovering, for each worker–establishment pair, the dates at which the employment relationship begins and ends within the year. We then take two reference dates 180 days apart—the end of the first quarter (day 90) and the end of the third quarter (day 270)—and define establishment employment at each date as the number of workers with an active spell at the establishment on that day, $E_{i,90}$ and $E_{i,270}$. A full-year (day 1 to day 365) window cannot identify worker flows, since genuine entries and exits at the endpoints cannot be distinguished from continuing spells. The six-month interior window avoids this problem. The establishment growth rate is $g_i = (E_{i,270} - E_{i,90})/\bar{e}_i$, with $\bar{e}_i = \frac{1}{2}(E_{i,90} + E_{i,270})$, which bounds g_i in $[-2, 2]$ in the usual way. Hires are workers with an active spell at day 270 but not at day 90 and separations are workers active at day 90 but not at day 270; both are expressed as a share of $\bar{e}_i$. By construction, net employment growth equals the hiring rate minus the separation rate. Establishments are sorted into 160 growth-rate bins. Panel (a) pools all workers; Panel (b) reports the relations separately for high- and low-skilled workers.

B.4 Sample Definition

The sample used in this study represents the intersection of the DADS and BRN datasets. Our analysis focuses exclusively on firms included in the BRN dataset, specifically those subject to the fiscal regime *réel normal*.[A6] In 2003, the dataset covered 24% of French firms, accounting for over 94.3% of total gross output (INSEE, 2004). We further exclude the NACE/ISIC Sections *'A - Agriculture, Forestry and Fishing'*, *'B - Mining and Quarrying'*, and *'K - Financial and insurance activities'*, which are under-represented in BRN. Moreover, by conditioning on DADS, we exclude all firms that do not report employees in a given year, also excluding firms that are still reported as "active" but are de facto out of the market.

After this initial selection, we further reduce the dataset to two different samples for the purposes of the aggregation and the estimation exercises, as discussed below.

[A6]In 2007, the threshold was €763,000 for trade and real estate sector firms and €230,000 for firms operating in all other sectors INSEE (2004).

Table B.5: Aggregation Sample

	N. of Firms			
	BRN	DADS	Merge	Aggregation
Manufacturing [C]	82,045	136,503	70,589	56,679
Electricity, gas, water, and waste [D-E]	3,913	6,304	2,720	2,058
Construction [F]	87,596	181,847	60,340	39,726
Non-Financial Market Services [G-N]	467,433	776,275	321,802	213,130
Non Market Services [O-U]	40,314	282,268	31,809	20,103
Tot	681,299	1,383,197	487,259	331,696
	Employment			
Manufacturing [C]		3,174,555	2,997,247	2,951,833
Electricity, gas, water, and waste [D-E]		147,806	132,024	129,867
Construction [F]		1,134,715	874,090	789,821
Non-Financial Market Services [G-N]		7,071,001	6,091,202	5,793,353
Non Market Services [O-U]		3,829,527	646,125	602,613
Tot		15,357,604	10,740,688	10,267,487

Notes: The table reports the average number of firms (top panel) and average employment (bottom panel) by sector over 1998–2007. Columns one and two give counts in BRN and DADS; the third column conditions on firms present in both datasets; the fourth gives the final aggregation sample, which retains only firms reporting both high- and low-skilled workers and for which equipment stock can be constructed or imputed (see Sections B.4 and C.1.6).

B.4.1 Aggregation Sample

In the sample used for the aggregation, we include only firms that report a non-missing number of both high- and low-skilled workers. However, we retain firms with missing values of equipment payments, for which we impute values using the approach presented in Section C.1.6.

In Table B.5 we provide details about the coverage of the dataset used for the aggregation. The first two columns of the first panel report the average number of firms, respectively, in BRN and DADS, by sector. The third column reports sector-specific statistics obtained using only firms covered in both datasets. Finally, in the fourth column, we report the coverage of the final dataset, which is obtained by excluding firms that do not report either low- or high-skilled workers and a small number of firms for which it is not possible to impute the value of equipment stock (see Section C.1.6). Similarly, the second panel reports statistics on total employment, apart from the first column since the employment data are retrieved from DADS and not BRN.

B.4.2 Estimation Sample

The sample used to estimate the micro-elasticities is smaller, as we only focus on firms that are continuously active over a 5-year period and impose further restrictions based on the specific features of the model used. In particular, the baseline estimates presented in equation (25) are based on a sample of importers, excluding firms with outlying values

Table B.6: Estimation Sample

	N. of Firms			
	Tab 3, Col. (2)	Tab 3, Col. (4)	Tab 4, Col. (1)	Tab 5, Col. (4)
Manufacturing [C]	12,190	4,622	9,206	18,691
Electricity, gas, water, and waste [D-E]	138	44	112	610
Construction [F]	507	370	429	8,067
Non-Financial Market Services [G-N]	14,064	8,034	6,991	35,852
Non Market Services [O-U]	155	83	110	3,499
Tot	27,053	13,153	16,848	66,718
	Employment			
Manufacturing [C]	1,712,116	412,496	1,554,792	1,841,865
Electricity, gas, water, and waste [D-E]	9,480	3,209	24,671	86,317
Construction [F]	54,977	35,338	99,067	379,584
Non-Financial Market Services [G-N]	971,668	436,358	1,231,955	2,988,024
Non Market Services [O-U]	26,232	8,112	38,195	237,808
Tot	2,774,474	895,514	2,948,681	5,533,597

Notes: The table reports the average number of firms (top panel) and average employment (bottom panel) by sector for the samples used to estimate the baseline specifications in Tables 3, 4, and 5. Each column corresponds to a distinct estimation sample, defined by the import, instrument, and outlier restrictions detailed in Section B.4.

of the trade-shock instruments used in the estimation. Instruments based on trade shocks result in outliers for some firms that trade a limited range of specialized products to small destinations. Following Aghion et al. (2024), we address this issue by regressing shocks (both in our WES and RER instruments) on a firm fixed effect and then truncate observations with residuals beyond the 99th and 1st percentiles. Thus, observations with the largest deviations in shocks (relative to their firm mean) are excluded from our sample.

In Table B.6, we report the average number of firms and employment by sector for the samples used to estimate the baseline model reported in Tables 3, 4, and 5. In the first column, we report the sector breakdown of the average number of firms and employment of the sample used in our baseline estimates of equation (25). The sample is limited to firms that import at least 1% of their gross output (on average over the sample period) and do not report anomalous values in the adopted instruments.[A7] In the second column, we impose a further restriction by excluding firms that import from any country to which they also export more than 10% of their total export value over the period. In the third column, we report sector-specific statistics for the sample used to estimate equations (26) and (27). This sample is limited to firms that import equipment products and survive the same outlier cleaning routine adopted for the other import instruments.[A8] Finally, in the

[A7]The instrument exploits differences in firm-specific import exposure across source countries and changes in real exchange rates between France and its trading partners.

[A8]As explained in Section 4.2, we use the real-exchange-rate instrument, the skill-specific wage instruments, and a world-export-supply instrument. The sample coincides with the one used to estimate column fourth column, we report the sector breakdown for the sample used to estimate equations (C.8) and (C.9). In this case, the sample is significantly larger, as the model relies only on the skill-specific wage instruments.

C Empirical Appendix

In this appendix we provide a detailed account on how we construct the firm-level capital stock for each asset category and our measure of firm- and skill-specific wages. Additionally, we provide additional details on the construction of the instrumental variables used in the estimation of the micro-elasticities, and our strategy to estimate the nested CES aggregate production function.

C.1 Measuring Stock and User Cost of Equipment

The data used in building our measure of the stock of equipment come from the BRN and the customs datasets. In 2003, BRN covered 24% of French firms, accounting for over 94.3% of total gross output (INSEE, 2004) and over 90% of the aggregate value of trade flows in customs records (authors' computation).

C.1.1 Equipment Imports

For the imported part of equipment investment, we rely on customs data, which provide information on partner country, CN8 product, firm, and year-specific transaction values and quantities (in kilograms). The initial dataset includes over 28 million import and 24 million export transactions, involving 551,008 French firms, 13,901 different CN8 products, and 190 partner countries.

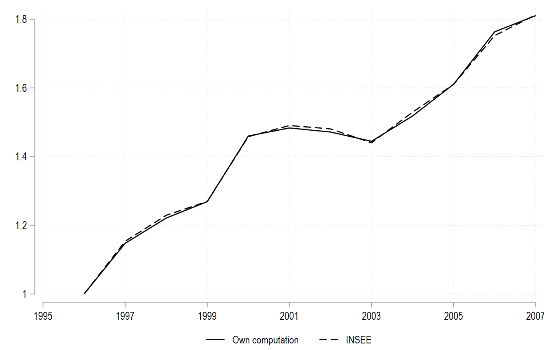
Unit-value prices are defined as the ratio between the value and quantity (typically measured in kilograms) associated with each transaction. Since certain firms had the option to report the number of units transacted instead of the quantity in kilograms, 5% of firms report missing values for the import quantity variable and 3.6% for the export quantity. However, they all report the units sold or purchased. We impute the missing

4 of Table 5, which restricts the nested-CES estimation to the CRESH/CXES sample for sample comparability. The two estimations differ in their instruments: the nested-CES estimation uses only the skill-specific wage instruments, while the CRESH estimation additionally uses the real-exchange-rate and world-export-supply instruments.

Figure C.1: Aggregate Trends

(a) Imports

(b) Exports



Notes: The figure compares the aggregate import and export trends computed from our cleaned customs data against the corresponding series reported by INSEE. The close correspondence between the two series is a check on the representativeness of our data (see Section C.1.1).

values using CN8-year-level quantity-unit ratios. This exercise reduces the share of missing values to 1.9% for imports and 1.4% for exports.

As explained in Section B.1, the number of variables gathered within each statistical framework depends on thresholds specific to the firm and the transactions. Firms are required to report individual transactions with non-European countries when they exceed €1,000. Moreover, intra-EU transactions are subject to an aggregate annual threshold that grows over the study period from €38,000 to €150,000. These features of the dataset create two potential issues. First, the increase over time of the minimum threshold for reporting intra-EU transactions could bias our results. Second, firms reporting under-threshold transactions might be different from firms that do not. To address these issues, we drop extra-EU transactions below the threshold of €1,000 and all transactions with European partners where the cumulative annual flow is below the threshold of €150,000. After these cleaning steps, we are left with 82% of the original imports, and 93% of the original exports, corresponding, respectively, to 99.4% and 99.7% of the total value.

In Figure C.1, we conduct a simple validation exercise, comparing the aggregate import/export trends produced with our data with those reported by INSEE. The high correlation between the two series provides reassurance regarding the representativeness of our data.

The creation of the import variables follows two steps. First, we compute a constant

CN8 product classification for the period 1996-2007 (hereafter "CN8plus") by implementing the method proposed by Van Beveren et al. (2012). Subsequently, we create a constant HS6 product classification (hereafter "HS6plus"), so that each CN8plus corresponds to a single HS6plus code. Finally, we link HS6plus product codes to (overlapping) macro-product categories: equipment, intermediate inputs and capital inputs, using the BEC product classification. In particular, we define equipment products as the ones that belong to the following categories of the BEC Rev. 4 classification: *Capital goods (except transport equipment), and parts and accessories thereof*; *521 - Transport equipment, other, industrial*; and *51 - Transport equipment, passenger motor cars*. The second step consists in excluding from the equipment variable all transactions with product category belonging to the macro-groups 'raw materials' (HS 01-15, 25-27, 31, 41) and 'services' (HS 97-99). Finally, we aggregate all import variables at the firm-country-product-year level.

C.1.2 Domestic Equipment Investments

We do not observe equipment investment directly in the BRN data and instead recover it from the variables that accounting rules report: the gross book-value stock of equipment (K^e_{it}), its accumulated depreciation (D^e_{it}), and a measure of total disposed assets (U_{it}), all defined as in Section B.2. The construction inverts the accounting identities that link these variables. The firm's *net* book value of equipment, $K^e_{it} - D^e_{it}$, rises with new investment and falls with depreciation and with the value of any equipment sold or scrapped during the year; solving that relationship for the investment flow, under straight-line depreciation over a useful life of T years, gives our proxy:

$$I^e_{it} = \frac{T}{T-1} \cdot \left[(K^e_{it} - K^e_{it-1}) - (D^e_{it} - D^e_{it-1}) + \sum_{s=t-T}^{t-1} \frac{I^e_{is}}{T} + \psi_{it} + U^e_{it} - \lambda_{it-1} \right],$$

where the auxiliary terms are defined as

$$U^e_{it} = U_{it} \frac{K^e_{it-1}}{K_{it-1}},$$

$$\psi_{it} = \begin{cases} \min\left\{ \frac{K^e_{i0}}{T},\ K^e_{i0} - D^e_{i0} - \frac{K^e_{i0}}{T}(t-1) \right\}, & \text{if } K^e_{i0} - D^e_{i0} - \frac{K^e_{i0}}{T}(t-1) > 0, \\ 0, & \text{otherwise,} \end{cases}$$

$$\lambda_{it} = \begin{cases} 0, & \text{if } t = 1, \\ \min\left\{ U^e_{i1},\ \psi_{i1} + \frac{I^e_{i1}}{T} \right\}, & \text{if } t = 2, \\ \min\left\{ U^e_{it-1} + \max\left\{ U^e_{it-2} - \sum_{s=t-T-1}^{t-2} \frac{I^e_{is}}{T}, 0 \right\}, \sum_{s=t-T}^{t-1} \frac{I^e_{is}}{T} \right\}, & \text{if } t > 2. \end{cases}$$

The year-on-year change in net book value, $(K^e_{it} - K^e_{it-1}) - (D^e_{it} - D^e_{it-1})$, removes the depreciation that straight-line accounting charges even when no capital is sold, which we add back over the trailing T vintages through $\sum_{s=t-T}^{t-1} I^e_{is}/T$. The term ψ_{it} captures the depreciation run-off of the initial stock K^e_{i0}, which predates the sample and is therefore missing from the trailing sum. The term U^e_{it} adds back the value of equipment disposed of during the year, apportioned from total disposals by equipment's lagged capital share and netted of the correction λ_{it-1}, which prevents assets that are simultaneously retired and fully depreciated from being added back twice. The factor $T/(T-1)$ adjusts for within-year depreciation of year-t investment. Because the construction relies on lagged values, our investment series begin in 1994. We validated the proxy against the actual investment figures available only to INSEE staff and obtained a correlation above 0.9.[A9]

Finally, we compute domestic equipment investment as the difference between our proxy for overall equipment investment and the value of imported equipment retrieved from customs data: $I^{e^{dom}}_{it} = I^e_{it} - I^{e^{for}}_{it}$.

C.1.3 Firm-Level Price Indices for Equipment Investment

Prices for imported equipment products are defined as unit values at the firm-, year-, product-, and country of origin-level. For domestic investment, we do not have firm-level information on prices. Therefore we exploit sector-, asset-, and year-specific data provided by KLEMS (see Section B.2).

At this point, we have annual firm-level data on domestic equipment investments and imported equipment investments at the level of product-origins, together with their prices. We next compute a firm-level price index for equipment to deflate the investment values and obtain the quantity of equipment investment.

Our index is a slight variation on the standard Sato-Vartia price index. We treat the domestic investment as a single equipment product originating from France, and define the share of equipment product k coming from the country of origin c as $S^e_{ckit} = I^e_{ckit}/\tilde{I}^e_{it}$, where $\tilde{I}^e_{it}$ is the total firm-level equipment investment at time t on product-country vari-

[A9]We are grateful to Jocelyn Boussard for help with this analysis.

Table C.1: Correlation Table - Alternative Price Indices

ε	∞	**5.44**	**4.49**	**3.37**
∞	1			
5.44	0.977	1		
4.49	0.976	0.999	1	
3.37	0.948	0.975	0.982	1

Notes: The table reports the correlation matrix between our baseline firm-level equipment price index (a Feenstra-corrected index with $\varepsilon = \infty$) and alternative indices constructed using the estimates of the elasticity of substitution ε from Table 3.

eties common between $t-1$ and t. Similarly, we define the share of equipment products k coming from country of origin c at $t-1$ as $S^e_{ckit-1} = I^e_{ckit-1}/\tilde{I}^e_{it-1}$, where $\tilde{I}^e_{it-1}$ is the total firm-level equipment investment at time $t-1$ on common (product-country) varieties between $t-1$ and t. Using these shares, we define product-country weights ω^e_{ckit} as:

$$\omega^e_{ckit} = \begin{cases} \frac{S^e_{ckit} - S^e_{ckit-1}}{s^e_{ckit} - s^e_{ckit-1}} & \text{if } S^e_{ckit} \neq S^e_{ckit-1}, \\ S^e_{ckit} & \text{if } S^e_{ckit} = S^e_{ckit-1}, \end{cases}$$

where, as before, small-cap letters denote the logarithm of the corresponding variable. We then compute the firm-level Sato-Vartia log price indices as:

$$\mathbb{P}^e_{it} = \exp\left(\sum_{ck} (p_{cki,t} - p_{cki,t-1}) \frac{\omega^e_{ckit}}{\sum_{ck} \omega^e_{ckit}}\right).$$

Subsequently, we compute equipment investment quantities Q^e_{it} using equipment investment values (I^e_{it},) and the price indices computed in the previous stage: $Q^e_{it} = I^e_{it}/P^e_{it}$ where $P^e_{it} \equiv \prod_{\tau=0}^{t} \mathbb{P}^e_{i\tau}$.

We also construct an alternative price index which takes into account changes in import varieties:

$$\hat{\mathbb{P}}^e_{it} = \mathbb{P}^e_{it} \left(\frac{\Lambda_t}{\Lambda_{t-1}}\right)^{\frac{1}{\varepsilon-1}}$$

where $\left(\frac{\Lambda_t}{\Lambda_{t-1}}\right)^{\frac{1}{\varepsilon-1}}$ is the standard Feenstra (1994) variety correction term, which takes account of the entry and exit of varieties. If entering varieties are more attractive than exiting varieties, the share of common varieties in total expenditure will be smaller in period t than in period $t-1$ $\left(\frac{\Lambda_t}{\Lambda_{t-1}}^{\frac{1}{\varepsilon-1}} < 1\right)$, which reduces the price index (since $\varepsilon \geq 1$). In Ta-

Table C.2: Equipment Stock - Summary Statistics

	Mean	10p	Median	90p	sd	N firms
Manufacturing [C]	10,668	0	560	7,875	211,130	56,679
Electricity, gas, water, and waste [D-E]	43,110	0	1,215	22,823	572,287	2,058
Construction [F]	668	0	180	1,147	4,529	39,726
Non-Financial Market Services [G-N]	3,571	0	186	1,893	164,174	213,130
Non Market Services [O-U]	1,331	0	192	1,911	13,800	20,103
Tot	4,538	0	221	2,480	164,967	331,696

Notes: The table reports summary statistics on equipment stock by sector over the sample period. The equipment stock is constructed as described in Section C.1.4. The final column reports the number of firms.

ble C.1, we report the correlation matrix between our baseline variable (equivalent to a Feenstra-corrected variable with $\varepsilon = \infty$) and alternative measures constructed with estimates of the elasticity of substitution from Table 3.

C.1.4 Construction of the Equipment Stocks

Finally, we compute the equipment stock X_{eit}, applying the perpetual inventory method:

$$X_{eit} = \begin{cases} \frac{1}{\delta^e_{st}} \frac{\sum_{\tau=0}^{T_i-1} Q^e_{i\tau}}{T_i}, & \text{if } t = 1, \\ (1-\delta^e_{st})X_{ei,t-1} + Q^e_{it}, & \text{if } t > 1, \end{cases}$$

where δ^e_{st} is a sector-specific depreciation rate obtained from the EU-KLEMS dataset. Following Mueller (2008), we initialize the capital stock of each firm i as the average investment over all T_i years in which the firm is observed in the sample, divided by the depreciation rate. To reduce the sensitivity of the equipment stock to this initialization strategy, we set the first two years to missing. As a result, the first year in which the stock is available is 1996. For most firms, we observe at least two years of data before our sample period, so this strategy does not affect coverage. However, the coverage is reduced for firms that enter during our sample period.[A10]

In Table C.2 we report the summary statistics for our proxy of equipment stock.

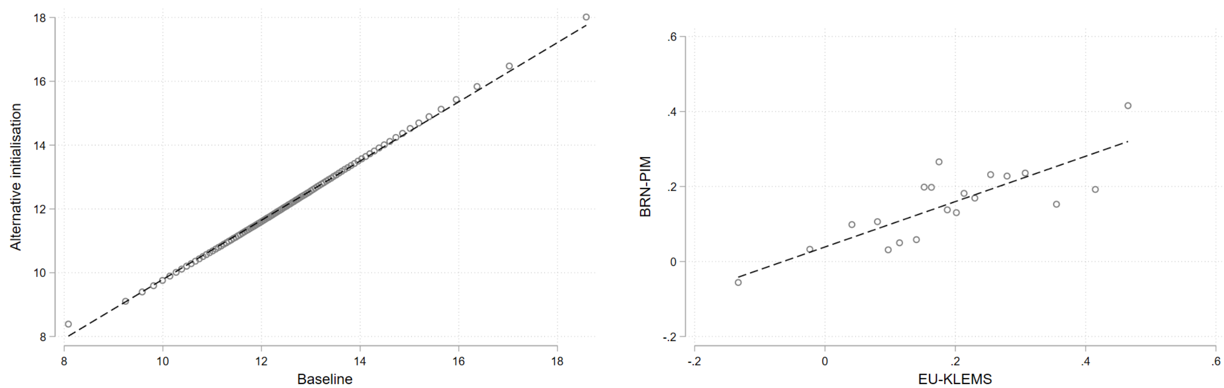
For the construction of our baseline equipment stock variable, we rely on an initial value of the capital stock computed using the dual of the price index for equipment investment defined in Section C.1.3. This approach maximizes internal consistency with the overall strategy used to construct the variable. An alternative approach would be to ini-

[A10]Moreover, we remove anomalous values in the computed equipment stocks: negative values, permanent falls by a factor of 10, one-off (yearly) increases or decreases by a factor of 50.

tialize the process with the book value of equipment reported in firms' balance sheets. Although book values are less precise for our purposes, we construct this alternative equipment stock variable and compare it with our baseline measure. The two equipment stock variables exhibit a correlation of 0.95 and differ only slightly at the extremes of the distribution, as shown in Panel (a) of Figure C.2. Panel (b) of Figure C.2 compares our measure aggregated at the sectoral level against a national-accounts benchmark taken from EU-KLEMS (2016 release). Across sectors, the 5-year change in our aggregated PIM stock co-moves strongly with the corresponding change in EU-KLEMS.

Figure C.2: Equipment Stock

(a) Alternative Initial Value (b) Correlation with EU-KLEMS



Notes: "PIM" is the firm-level perpetual-inventory stock we construct, built from firm-level equipment investment deflated with our firm-specific equipment price index and accumulated under geometric depreciation with the initialization of Mueller (2008). "EU-KLEMS" is the chain-linked volume fixed capital stock, at 2010 prices, taken from the EU-KLEMS 2016 release, which sources the series from Eurostat. Panel (a) compares our baseline PIM measure with an alternative PIM measure initialized using the book value of equipment reported in firms' balance sheets. Panel (b) plots, across sectors, the 5-year log change in our aggregated PIM stock against the corresponding change in EU-KLEMS.

C.1.5 Construction of the User Cost of Equipment

We define the firm-level user cost of equipment W_{eit} (the effective rental price of equipment capital) as follows:

$$W_{eit} = P^e_{it}\left(R^e_t + \delta^e_{st} - \frac{p^e_{s,t+1} - p^e_{s,t-2}}{3}\right)$$

where P^e_{it} is the firm-specific price of equipment products purchased by firm i in time t defined above, R^e_t is the required rate of return on equipment investments, and p^e_{st} and δ^e_{st} are the log price and depreciation rate of equipment products at the sector level retrieved respectively from Eurostat and EU-KLEMS (see Section B.2). Finally, we obtain firm-level payments to capital equipment by multiplying the user cost of equipment W_{eit} by the measure of equipment stock X_{eit}.

C.1.6 Imputation

As a final step, we develop a simple imputation strategy to reduce the number of observations reporting missing values for the equipment payments variable. A large share of these missing values are a direct consequence of the cleaning procedures implemented. In particular, the procedures described in Section C.1.2 and C.1.4 to compute the equipment investment and the equipment stock proxies produce missing values in the first three years in which a firm is observed. In our final sample, we do not observe equipment stock for around 30% of firm-year observations and we cannot construct our variable for equipment payments.[A11]

To address this issue, we impute the expenditures on equipment for these firms based on industry-specific relationships between factor payments and size. Specifically, we run the following regression:

$$\log(W_{eit}X_{eit}) = \delta_s^y y_{it} + \delta_s^n n_{it} + \psi_{st} + v_{eit}$$

where $W_{eit}X_{eit}$ stands for the payments to equipment factors by firm i in industry s in year t; y_{it} is the logarithm of gross output, and where n_{it} is the logarithm of employment (number of hours). We implement this imputation procedure on all observations reporting a reliable measure of gross output and this allows us to increase the coverage from 69.3% to 91.4%.[A12]

We use these imputed values to extend the sample of firms used in the aggregation exercise performed in Section 5, which only requires factor payments when studying the case of a uniform shock in the rental price of equipment. Figure D.5 shows that this imputation strategy leaves the main quantitative results of the paper almost unaffected. To avoid working with different aggregation samples, when studying the case of heterogeneous shocks we simply assign the average aggregate rental price of equipment to the imputed firms. While this approach reduces heterogeneity in our sample, the imputed

[A11]Specifically, 15.6% of missing values are due to the initialization of the investment proxy, 32.4% are due to the initialization of the perpetual inventory method and 24.6% are caused by other cleaning steps and 27.5% are due to firms not reporting data on BRN in a given year.

[A12]The remaining firm-year observations report neither equipment stock nor gross output. As discussed in Section B.4, our sample is obtained by intersecting the DADS and BRN datasets. However, some firms appear in DADS for more years than in BRN. This discrepancy may be due to the fact that they did not always meet the minimum threshold to be subject to the ‘réel normal’ fiscal regime.

firms are typically smaller and less likely to import capital equipment. So this assumption is not entirely implausible and these firms are essentially treated as those that purchase capital equipment only domestically, for which we cannot observe a firm-specific price. However, we refrain from using imputed values in the estimation of the micro-elasticities, which requires reliable values of both quantities and rental prices of capital equipment.

C.1.7 Aggregate Equipment Shocks

In this section, we outline the methodology used to construct the uniform equipment shocks employed in the analysis. In the baseline aggregation exercise (Table 8 and Figure 3b), the shock is defined as the log changes in the aggregate rental price based on industry-level data. First, we construct the industry-specific user cost of equipment, W_{est}, following the same approach detailed in Section C.1.5 with firm-level data:

$$W_{est} = P^e_{st}\left(R^e_t + \delta^e_{st} - \frac{p^e_{s,t+1} - p^e_{s,t-2}}{3}\right)$$

where P^e_{st} is the industry s price index for the ESA 2010 asset category *N110G - Machinery, Equipment and Weapons* retrieved from Eurostat, R^e_t is the required rate of return on equipment investments,[A13] and δ^e_{st} is the industry-specific depreciation rate obtained from EU-KLEMS.[A14] Subsequently, we aggregate across industries using the revised Sato-Vartia price aggregation method described in Section C.1.3:

$$W_{et} = \exp\left(\sum_s \left(\log W_{est} - \log W_{est-1}\right)\frac{\omega^e_{st}}{\sum_s \omega^e_{st}}\right).$$

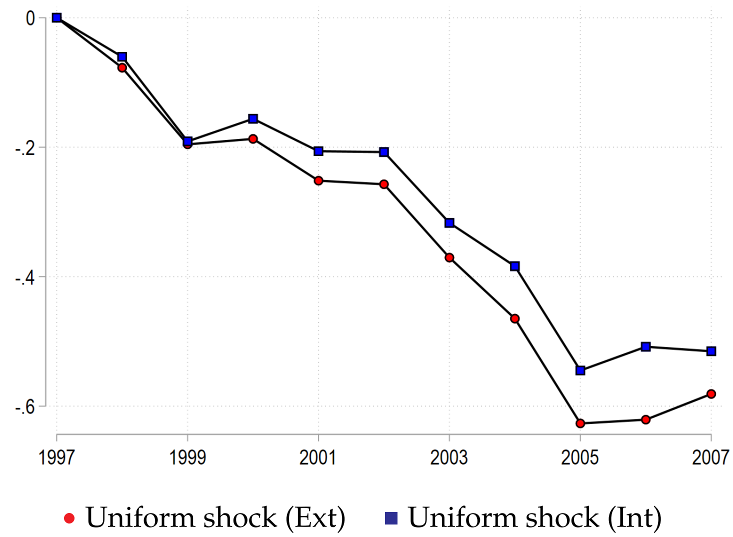
Lastly, we define the aggregate uniform equipment shock as the difference between the year-on-year log change in the aggregate user cost of equipment and the log change in the minimum wage (Smic) $\underline{W}_t$ retrieved from INSEE:

$$d\log \widehat{W}_{et} = d\log W_{et} - d\log \underline{W}_t$$

To facilitate a better comparison with the heterogeneous price shocks in Table D.9 and Figure D.1, we also construct an alternative uniform shock obtained by aggregating the firm-level heterogeneous shocks to the user cost of equipment. Let $\widetilde{\Lambda}_{eit}$ denote firm i's share of aggregate equipment payments as follows, using the time-indexed analogue of

[A13] We use the annual average of France 10-year Government bond Yield, as retrieved from FED St. Louis.
[A14] See Section B.2 for more details on the two data sources.

Figure C.3: Uniform Equipment Shocks



Notes: The figure plots the cumulated equipment shocks observed over the period, rescaled by the aggregate change in the low-skilled wage proxied with the change in the nominal hourly minimum wage sourced from the French Ministry of Labor. The uniform shock represented with red circles is computed as the log change in the rental price using industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. The uniform shock represented with blue squares is computed by aggregating the firm-level heterogeneous changes in the user cost of equipment (W_{eit}).

equation (14):

$$\widetilde{\Lambda}_{eit} = \frac{1}{2}\left(\Lambda_{it}\widehat{\theta}_{eit} + \Lambda_{it-1}\widehat{\theta}_{eit-1}\right) = \frac{1}{2}\left(\frac{W_{eit}X_{eit}}{\sum_i W_{eit}X_{eit}} + \frac{W_{eit-1}X_{eit-1}}{\sum_i W_{eit-1}X_{eit-1}}\right)$$

where $W_{eit}X_{eit}$ and $W_{eit-1}X_{eit-1}$ stand for the payments to equipment factors measured in t and $t-1$ for firms i that are active in the market in both years.

The alternative uniform shock is defined as:

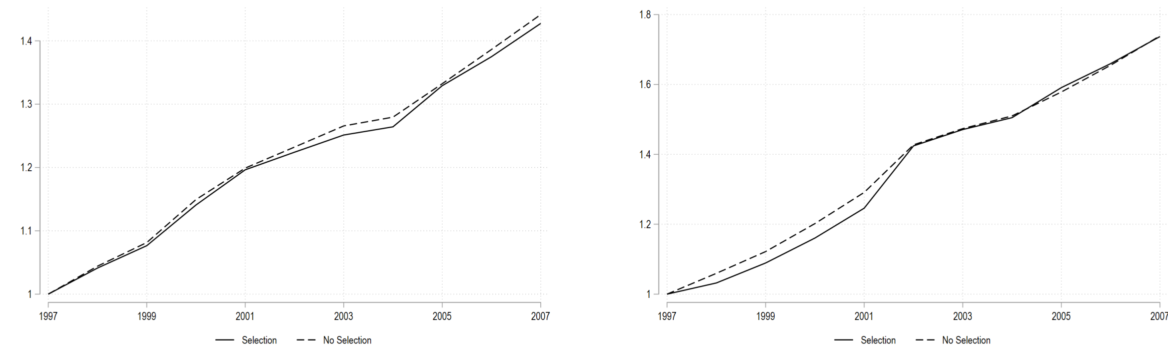
$$d\log \overline{W}_{et} = \left(\sum_i \widetilde{\Lambda}_{eit} d\log W_{eit}\right) - d\log \underline{W}_t$$

See Figure C.3 for a comparison between the two aggregate uniform shocks.

In the comparison exercise of Section 5.3, we define aggregate equipment payments as the sum of firm-level equipment payments ($\sum_i W_{eit}X_{eit}$). We further apply a quality adjustment to the aggregate price of equipment to match the exercise in Krusell et al. (2000). We take the average annual difference between the official NIPA series and the TORN series computed by Krusell et al. (2000) and redefine our aggregate uniform equipment

Figure C.4: Aggregate trends

(a) Low-skilled (b) High-skilled



Notes: The figure plots the aggregate trend in observed hourly wages for low-skilled and high-skilled workers, computed with and without the worker-level selection criteria described in Section C.2.1.

shock as:

$$d \log \widetilde{W}_{et} = d \log W_{et} - 0.033$$

We then index $\widetilde{W}_{et}$ to one in 1997 and define the aggregate equipment stock as:

$$\widetilde{X}_{et} = \frac{\sum_i W_{eit} X_{eit}}{\widetilde{W}_{et}}.$$

C.2 Measuring Firm- and Skill-Specific Wages and Employment

In the model, all workers within a skill group are identical. Consequently, we need to correct the observed changes in firm- and skill-specific wages for worker-level unobservables and changes in returns to worker observable characteristics.

C.2.1 Data Cleaning

Our model-consistent measures of firm- and skill-specific wages are computed using data retrieved from DADS Poste. This dataset provides, for each worker, the observed wage in t and $t-1$, as well as the number of hours worked, the duration of the employment contract, and a set of individual characteristics.[A15] To construct reliable proxies for high- and low-skilled wages, we first exclude "atypical workers," namely those younger than

[A15]The dataset is at the job-spell level. Starting in 2002, a unique worker identifier is provided so that job spells can be correctly aggregated at the worker level. Before 2002, we cannot implement this aggregation and consider each spell as a separate worker.

Table C.3: Correlation Table - Wage Proxies

Low-skilled			
	Occupation, sex, full-time status	Sex, full-time status	No controls
Occupation, sex, full-time status	1		
Sex, full-time status	0.999	1	
No controls	0.997	0.998	1
High-skilled			
	Occupation, sex, full-time status	Sex, full-time status	No controls
Occupation, sex, full-time status	1		
Sex, full-time status	0.999	1	
No controls	0.994	0.995	1

Notes: The table reports, separately for low- and high-skilled workers, the correlation between our baseline firm-and-skill-specific wage measure (controlling for two-digit occupation, sex, and full-time status) and two alternatives controlling for sex and full-time status only, and for nothing at all. See Section C.2.2 for the estimation of the wage residuals.

18 or older than 75, cross-border workers, interns and trainees ("apprentis," "stagiaires," and "emploi aidés"). We also exclude workers employed for less than a month or who recorded fewer than 150 paid hours.[A16] Finally, we exclude all workers for whom occupational information is missing.

Worker-level hourly wages are computed by dividing individual wage bills by the reported number of hours worked. The resulting variable is winsorized between a minimum value equal to 80% of the hourly minimum wage and €1,000.

In Figure C.4, we report the aggregate trend of the observed hourly wage variables, both with and without the selection strategy discussed above.

C.2.2 Estimation of Wage Residuals

In order to compute our residual wage variable, we estimate for both high- and low-skilled workers a Mincerian regression on log wage changes between $t-1$ and t controlling for worker observables (dummy variables for sex, full-time status and 2-digit occupation categories) and firm-year fixed effect. The specification takes the following form:

$$wage_{ji,t} - wage_{ji,t-1} = \boldsymbol{b}_t' \boldsymbol{X}_{jit} + \gamma_{fit} + e_{jit},$$

where worker j is classified as either $f = h$- or $f = \ell$- skilled depending on their occupation (*catégories socioprofessionnelles*) at time t, as defined earlier, and where $\boldsymbol{X}_{jit}$ denotes the vector of worker characteristics. The year-on-year change in the firm-specific wage is

[A16] In 2002, 23.5% of job spells fall below one of the two thresholds, but they account for only 1.3% of total hours worked.

the estimated firm-year fixed effect.

We also produce two alternative proxies, computed by estimating the same model controlling for sex and full-time status and with no controls at all. In Table C.3, we report the correlation tables between our baseline variables and these alternatives.

As a final step, we chain the year-on-year changes together to obtain five-year changes, so that $\Delta w_{fit} = \sum_{\tau=0}^{4} \widehat{\gamma}_{fi,t-\tau}$ with $f \in \{h, \ell\}$. We further measure five-year changes in employment, Δx_{hit} and $\Delta x_{\ell it}$, by deflating changes in firm-and-skill-specific wage bills by the corresponding five-year wage change, Δw_{hit} and $\Delta w_{\ell it}$.

C.2.3 Aggregate Skill Supply and Wages

To construct the aggregate skill premium and skill supply, we construct factor-specific aggregate wages that are not influenced by the compositional adjustment that occurred over the study period. Specifically, we group worker-level observations based on the same set of covariates used to estimate the residuals in the previous section (sex, full-time status, and 2-digit occupation categories). We then measure the (weighted) average wage within each group for both high- and low-skilled workers, and compute the weighted average across all g groups within skilled and unskilled categories, where the weight is defined in terms of number of hours worked and is fixed across time by taking an average over the sample period:

$$\widetilde{W}_{ft} = \sum_{g \in G_f} \frac{W_{fgt} X_{fgt}}{X_{fgt}} \frac{\overline{X_{fg}}}{X_f}$$

where $W_{fgt} X_{fgt}$ is the sum of observed payments (wage bills) to factor $f \in \{h, \ell\}$ within group g and X_{fgt} is the corresponding observed number of hours worked (with slight abuse of notation). The compositionally adjusted hours used to construct the relative skill supply are computed from the observed wage bills by skill and the compositionally adjusted wages defined above.

In the comparison exercise of Section 5.3, we compute aggregate factor shares from the observed firm-level labor payments (wage bills) and the equipment payments computed in Sections C.1.5 and C.1.6, as follows:

$$\theta_{ft} = \frac{\widetilde{W}_{ft}\widetilde{X}_{ft}}{\sum_{f'} \widetilde{W}_{f't}\widetilde{X}_{f't}},$$

where $\widetilde{W}_{et}$ and $\widetilde{X}_{et}$ are defined in Section C.1.7. To better match aggregate data, we further rescale the aggregate factor shares. In particular we target the labor income share for the total economy in 1997 from EU-KLEMS, and we rescale the aggregate factor shares θ_{ft} with the rescaling constants κ_n for $f \in \{\ell, h\}$ and κ_e for $f = e$, defined as:

$$\frac{\theta_{\ell,1997} + \theta_{h,1997}}{\kappa_n} = 0.622,$$
$$\frac{\theta_{e,1997}}{\kappa_e} = 0.378.$$

We then further rescale the factor shares in the years $t > 1997$ to ensure that they sum to one in every year.

C.3 Additional Details on the Construction of Instruments

Real Exchange Rate Instrument. In Section 3.3 we introduce a firm-specific real-exchange-rate instrument. This instrument leverages variations in firm-specific import exposure across different source countries, alongside changes in real exchange rates between France and its trading partners.

In Section 4.2 we describe a sector-specific version of this instrument. It is defined formally as

$$RER_{st} = \sum_{c} \frac{\sum_{\tau=5}^{6} M_{cs,t-\tau}}{\sum_{\tau=5}^{6} M_{s,t-\tau}} \Delta \log \left(NER_{ct} \cdot \frac{Defl_{FR,t}}{Defl_{ct}} \right), \tag{C.1}$$

where $\sum_{\tau=5}^{6} M_{cs,t-\tau} / \sum_{\tau=5}^{6} M_{s,t-\tau}$ is the pooled two-year pre-period import share of origin country c in sector s's total imports.

Skill-Specific Wage Instruments. The skill-specific-wage instruments described in Section 4.2 are defined formally as

$$\Delta SSW_{it}^{f,-s} = \sum_{r} \sum_{s \neq s_i^r} \left(\frac{\sum_{\tau=5}^{6} L_{ni,t-\tau}^{r}}{\sum_{\tau=5}^{6} L_{ni,t-\tau}} \right) \cdot \left(\frac{\sum_{\tau=5}^{6} L_{fs,t-\tau}^{r}}{\sum_{\tau=5}^{6} L_{f,t-\tau,-s_i^r}^{r}} \right) \cdot \Delta \log GO_{st}, \tag{C.2}$$

for $f \in \{h, \ell\}$. Here, $\Delta \log GO_{st}$ is the five-year change in gross output for sector s at the national level in France. Exposure shares are constructed as the product of two components: (1) firm i's employment in region r, $\sum_{\tau=5}^{6} L^{r}_{ni,t-\tau}$, relative to its total employment across all domestic regions, $\sum_{\tau=5}^{6} L_{ni,t-\tau}$; (2) region r employment of skill-type f in sector s, $\sum_{\tau=5}^{6} L^{r}_{fs,t-\tau}$, relative to region r total employment of skill-type f across all sectors, excluding firm i's own sector in region r, $\sum_{\tau=5}^{6} L^{r}_{f,t-\tau,-s^{r}_{i}}$.

The intuition is that a rise in the nationwide level of output of a sector s that is particularly skill- (or unskill-) intensive will raise the skilled (unskilled) wage in a region with a large share of employment in that sector, affecting firms based on their spatial distribution over the territory.

For these instruments, we exploit gross output data retrieved from EU-KLEMS. The choice to use this external variable instead of aggregating firm-level gross output is primarily explained by the higher stability of aggregate data. Moreover, while BRN covers only firms subject to the 'réel normal' fiscal regime,[A17] EU-KLEMS data cover the whole economy and thus represent a better proxy of the sectoral variation in skill-specific-demand.

World-Export-Supply Instrument. For each exporting source country c and HS6 product k in the set of intermediate products $\mathcal{K}^{I}$, we measure the value of total exports toward all countries excluding France in year t, denoted by Exp_{ckt}. We use this variable to construct a firm-i-specific average of changes in export supplies weighting by initial intermediate product and source country import shares. We refer to this as the world-export-supply instrument—as in Hummels et al. (2014)—and define it formally as

$$\Delta WES_{it} = \sum_{c} \sum_{k \in \mathcal{K}_{I}} \frac{\sum_{\tau=5}^{6} M^{I}_{cki,t-\tau}}{\sum_{\tau=5}^{6} M^{I}_{i,t-\tau}} \Delta \log Exp_{ckt}, \tag{C.3}$$

Here, $\Delta \log Exp_{ckt}$ is the five-year change in the logarithm of exports of intermediate product k from source country c to the rest of the world (excluding France). Exposure shares are constructed as firm i's import value of intermediate good k from country c over years $t-5$ and $t-6$, $\sum_{\tau=5}^{6} M^{I}_{cki,t-\tau}$, relative to firm i's total imports of intermediate goods over the same two-year window, $\sum_{\tau=5}^{6} M^{I}_{i,t-\tau}$.

The intuition is that an increase in productivity for source country c in producing intermediate product k will reduce the marginal cost of a firm i for which this country-product pair constitutes a large share of intermediate imports.

[A17] see Section B.1 for more details.

C.4 Additional Details on Demand Estimation

Denote by $\Delta\widetilde{p}_{it}$ and $\Delta\widetilde{p}_{st}$ the log change in prices at fixed firm-level productivity. We have

$$\Delta\widetilde{p}_{it} = \Delta\widetilde{c}_{it} \approx \sum_{f\in\{\ell,h,e\}} \tfrac{1}{2}\left(\theta_{fi,t} + \theta_{fi,t-5}\right)\Delta w_{fi,t}, \tag{C.4}$$

$$\Delta\widetilde{p}_{st} = \Delta\widetilde{c}_{st} \approx \sum_{i\in\mathcal{I}_s} \tfrac{1}{2}\left(\Lambda_{i|s,t} + \Lambda_{i|s,t-5}\right)\Delta\widetilde{c}_{it}, \tag{C.5}$$

where, as before, θ_{fit} stands for the share of factor-f in firm-i costs at time t, and where $\Lambda_{i|s,t}$ stands for the share of firm-i in total sector-s costs at time t. We estimate equation (25) replacing firm and sector-level price changes with these changes in prices at fixed productivities. As described in the text, this introduces an additional component into the error term corresponding to firm-level productivity changes and their aggregation within sector.

C.5 Estimating Equations with Firm-Level CXES Production Function

Equations (26) and (27) are derived under the CRESH functional form. Here we derive analogous estimation equations under the constant cross-factor elasticities of substitution restriction $\sigma_{ff',i} \equiv \zeta_{ff'}$ (CXES) introduced in Example 4 of Section 2.3.

From the definitions of the firm-level cross-elasticities in (11), the changes in firm i's log demands for low- and high-skilled labor at fixed output satisfy

$$\Delta x_{\ell it} = \zeta_{\ell h}\,\theta_{hit}(\Delta w_{hit} - \Delta w_{\ell it}) + \zeta_{\ell e}\,\theta_{eit}(\Delta w_{eit} - \Delta w_{\ell it}) + \Delta y_{it} + \widetilde{\nu}_{\ell it},$$
$$\Delta x_{hit} = -\zeta_{\ell h}\,\theta_{\ell it}(\Delta w_{hit} - \Delta w_{\ell it}) + \zeta_{he}\,\theta_{eit}(\Delta w_{eit} - \Delta w_{hit}) + \Delta y_{it} + \widetilde{\nu}_{hit}.$$

By Allen–Uzawa symmetry and homogeneity of factor demand of degree zero, equipment demand admits two equivalent forms, one for each choice of skill group as the reference factor:

$$\Delta x_{eit} = \zeta_{he}\,\theta_{hit}(\Delta w_{hit} - \Delta w_{\ell it}) - (\zeta_{\ell e}\,\theta_{\ell it} + \zeta_{he}\,\theta_{hit})(\Delta w_{eit} - \Delta w_{\ell it}) + \Delta y_{it} + \widetilde{\nu}_{eit},$$
$$\Delta x_{eit} = -\zeta_{\ell e}\,\theta_{\ell it}(\Delta w_{hit} - \Delta w_{\ell it}) - (\zeta_{\ell e}\,\theta_{\ell it} + \zeta_{he}\,\theta_{hit})(\Delta w_{eit} - \Delta w_{hit}) + \Delta y_{it} + \widetilde{\nu}_{eit}.$$

Subtracting the low- and high-skilled equations from the first and second equipment-demand forms, respectively, and using the same equipment-demand form to substitute out $(\Delta w_{hit} - \Delta w_{\ell it})$ gives

$$\Delta x_{eit} - \Delta x_{\ell it} = \frac{\zeta_{he} - \zeta_{\ell h}}{\zeta_{he}}(\Delta x_{eit} - \Delta y_{it}) - \Psi_{\ell it}(\Delta w_{eit} - \Delta w_{\ell it}) + \nu_{\ell it},$$
$$\Delta x_{eit} - \Delta x_{hit} = \frac{\zeta_{\ell e} - \zeta_{\ell h}}{\zeta_{\ell e}}(\Delta x_{eit} - \Delta y_{it}) - \Psi_{hit}(\Delta w_{eit} - \Delta w_{hit}) + \nu_{hit},$$

where the wage coefficients are

$$\Psi_{\ell it} \equiv \Psi_{\ell}(\boldsymbol{\theta}_{it}; \boldsymbol{\zeta}) = \zeta_{\ell e}\, \theta_{eit} + \zeta_{\ell h}\, \theta_{hit} + \frac{\zeta_{\ell e}\, \zeta_{\ell h}}{\zeta_{he}}\, \theta_{\ell it},$$
$$\Psi_{hit} \equiv \Psi_{h}(\boldsymbol{\theta}_{it}; \boldsymbol{\zeta}) = \zeta_{he}\, \theta_{eit} + \zeta_{\ell h}\, \theta_{\ell it} + \frac{\zeta_{he}\, \zeta_{\ell h}}{\zeta_{\ell e}}\, \theta_{hit},$$

and the residuals are $\nu_{\ell it} \equiv (\zeta_{\ell h}/\zeta_{he})\, \widetilde{\nu}_{eit} - \widetilde{\nu}_{\ell it}$ and $\nu_{hit} \equiv (\zeta_{\ell h}/\zeta_{\ell e})\, \widetilde{\nu}_{eit} - \widetilde{\nu}_{hit}$. Substituting $\Delta y_{it} = \frac{\varepsilon}{\varepsilon - 1}\Delta r_{it}$ via the product-demand condition, as in the proof of equations (26) and (27), and absorbing the deterministic component of each residual into a time fixed effect β_{ft} yields

$$\Delta x_{eit} - \Delta x_{\ell it} = \beta_{\ell t} - \Psi_{\ell it}(\Delta w_{eit} - \Delta w_{\ell it}) + \left(\tfrac{\zeta_{\ell h}}{\zeta_{he}} - 1\right)\left(\tfrac{\varepsilon}{\varepsilon - 1}\Delta r_{it} - \Delta x_{eit}\right) + \nu_{\ell it}, \tag{C.6}$$
$$\Delta x_{eit} - \Delta x_{hit} = \beta_{ht} - \Psi_{hit}(\Delta w_{eit} - \Delta w_{hit}) + \left(\tfrac{\zeta_{\ell h}}{\zeta_{\ell e}} - 1\right)\left(\tfrac{\varepsilon}{\varepsilon - 1}\Delta r_{it} - \Delta x_{eit}\right) + \nu_{hit}. \tag{C.7}$$

The connection with the CRESH estimating equations is immediate. If, in the same algebra, the cross elasticities are given by the CRESH-implied firm-specific elasticities $\sigma_{ff',i} = \sigma_f \sigma_{f'}/\overline{\sigma}_i$, where $\overline{\sigma}_i \equiv \sum_g \sigma_g \theta_{git}$, then $\sigma_{\ell h,i}/\sigma_{he,i} = \sigma_\ell/\sigma_e$, $\sigma_{\ell h,i}/\sigma_{\ell e,i} = \sigma_h/\sigma_e$, $\Psi_{\ell it} = \sigma_\ell$, and $\Psi_{hit} = \sigma_h$. Thus the corresponding equations reduce to the CRESH estimating equations (26) and (27).

C.6 Estimating Equations with Firm-Level Nested CES Production

The production function specified in equation (17) implies that firm i's factor demand satisfies[A18]

$$\Delta x_{hit} - \Delta x_{\ell it} = -\varsigma\ (\Delta w_{hit} - \Delta w_{\ell it}) + \frac{\varsigma - \rho}{1 - \rho}\, \Delta \log\left(\frac{\theta_{hit}}{\theta_{eit} + \theta_{hit}}\right) + u_{\ell it}, \tag{C.8}$$
$$\Delta x_{hit} - \Delta x_{eit} = -\rho\ (\Delta w_{hit} - \Delta w_{eit}) + u_{eit}, \tag{C.9}$$

[A18] See the derivation based on the demand system in equations (A.3) and (A.4) in Appendix A.2.1.

where, as before, x_{fit} is the logarithm of employment of factor f in firm i at time t. The structural residuals $u_{\ell it}$ and u_{eit} depend on changes in firm-specific factor-augmenting productivities Δz_{fi}. All variables are measured as in the case of the CRESH specification discussed above.

C.7 Estimation of the Nested CES Aggregate Production Function

If we assume a nested CES aggregate production function, equations (A.3) and (A.4) suggest the following estimation equations

$$x_{ht} - x_{et} = -\rho \left(w_{ht} - w_{et} \right) + \nu_{1,t}, \tag{C.10}$$

$$x_{ht} - x_{\ell t} = -\varsigma \left(w_{ht} - w_{\ell t} \right) + \frac{\varsigma - \rho}{1 - \rho} \log \left(\frac{\theta_{ht}}{\theta_{et} + \theta_{ht}} \right) + \nu_{2,t}, \tag{C.11}$$

where the residuals on the right hand side are related to the factor-augmenting productivity terms

$$\nu_{1,t} \equiv \left(\rho - 1 \right) \left(z_{ht} - z_{et} \right),$$
$$\nu_{2,t} \equiv \left(\varsigma - 1 \right) \left(z_{ht} - z_{\ell t} \right).$$

In contrast to our approach, which is based on a combination of firm-level estimation of elasticities of production and demand together with theory-consistent aggregation, Krusell et al. (2000) rely on the identification assumption that $\nu_{1,t}$ and $\nu_{2,t}$ are the sum of a constant and a zero-mean residual that is uncorrelated with the regressors on the right hand side. Relying on the same identification assumption, we can jointly estimate the model parameters in equations (C.10) and (C.11) using a nonlinear GMM approach that uses the right-hand side variables as instruments.

D Additional Tables and Figures

Tables D.1, D.2, and D.3 revisit the estimation of the demand elasticities, ε and η, from Table 3.

Columns 1 and 2 of Table D.1 replicate columns 1 and 2 of Table 3. In column 3, we replicate the specification used in column 2 but using two-digit sectors instead of four-digit sectors. In column 4, we replicate column 3, but using one-digit sectors rather than two-digit sectors.

In Table D.2 we revisit the estimation of ε using 2SLS (column 1 is estimated using

Table D.1: Between-Firm Demand Elasticity Estimates (Alternative Aggregations)

	(1)	(2)	(3)	(4)
ε	0.75	4.49	5.31	2.92
	(0.01)	(0.77)	(0.99)	(0.30)
η	0.80	3.12	2.03	1.18
	(0.02)	(0.43)	(0.75)	(0.55)
Observations	162,320	162,320	158,796	162,245
Eta level	4 dig	4 dig	2 dig	1 dig
Year FE	Yes	Yes	Yes	Yes
Sel Costs		No	Yes	No
IV	-	*RER*	*RER*	*RER*
SW Min F-Stat		45.80	42.42	185.82

Notes: This table reports the results of estimating equation (25) by GMM. The dependent variable is the 5-year change in firm i's value added and the independent variables are total input costs at the firm and sectoral level, with sectors defined at the 4-digit level (columns 1 and 2), at the 2-digit level (column 3), or at the 1-digit level (column 4). All instrumented columns use the firm-level RER instrument defined in equation (24) and the sectoral RER instrument defined in equation (C.1). All variables are in deviations from year means in all columns. To avoid weak identification in column 3 we drop the bottom and top 1% of 5-year firm-level changes in total input costs (Sel Costs Yes). We report the minimum value of the Sanderson and Windmeijer (2016) first-stage F statistics obtained by running the equivalent 2SLS estimation. Robust standard errors are reported in parentheses.

Table D.2: Between-Firm Demand Elasticity Estimates (2SLS - ε only)

	OLS	2SLS				
	(1)	(2)	(3)	(4)	(5)	(6)
Δc	0.26	-3.96	-2.13	-3.97	-2.92	-1.93
	(0.01)	(0.91)	(0.36)	(1.13)	(0.66)	(0.31)
Observations	162,150	162,150	162,284	78,665	161,975	162,245
Year-Sector FE	4 dig	4 dig	2 dig	4 dig	4 dig	1 dig
Excl Exp Mkt	No	No	No	Yes	No	No
IV	-	*RER*	*RER*	*RER*	*RER+SSW*	*RER*
KP F-Stat	.	32.26	113.23	21.49	13.88	146.82
ε	0.74	4.96	3.13	4.97	3.92	2.93

Notes: This table reports the results of estimating equation (25) by 2SLS when the sector-level price changes are absorbed by sector-year fixed effects. The dependent variable is the 5-year change in firm i's value added and the independent variables are total input costs at the firm level. Columns 2-4 and 6 use the firm-level RER instrument defined in equation (24), column 5 additionally uses the skill-specific-wage instruments defined in equation (C.2). In column 4, we exclude firms that import from any country to which they also export more than 10% of their total export value over the period. We report the Kleibergen and Paap (2006) rk Wald (KP F stat) and robust standard errors in parentheses.

OLS), including sector-year fixed effects that absorb sector-level price changes, so that η is not estimated. Column 2 uses 4-digit sectors, as in our baseline estimation in Table 3. The resulting value of ε is slightly higher than in our baseline GMM estimation. Column 3 uses 2-digit sectors and the elasticity falls, as expected. In column 4 we exclude firms that import from any country to which they also export more than 10% of their total export value (averaged across years); this leaves our estimate essentially unchanged. In column 5 we include the two SSW instruments, in addition to the RER instrument; this substantially reduces first stage power. Finally, in column 6 we use one-digit sectors, and the resulting estimate falls, as expected.

Table D.3: Between-Firm Demand Elasticity Estimates (3 Nests)

	(1)	(2)	(3)	(4)
ε	0.75	**5.19**	6.37	3.92
	(0.01)	**(0.93)**	(1.69)	(0.58)
η	0.78	**3.14**	3.72	2.47
	(0.02)	**(0.48)**	(0.89)	(0.30)
Observations	162,320	**162,320**	78,918	162,149
Eta level	4 dig	**4 dig**	4 dig	4 dig
Year-1digSect FE	Yes	**Yes**	Yes	Yes
Excl Exp Mkt	No	**No**	Yes	No
IV	-	***RER***	*RER*	*RER+SSW*
SW Min F-Stat		**39.39**	16.45	23.10

Notes: This table reports the results of estimating equation (25) by GMM, with all variables in deviations from year by 1-digit sector means. The dependent variable is the 5-year change in firm i's value added and the independent variables are total input costs at the firm and 4-digit sectoral level. All instrumented columns use the firm-level RER instrument defined in equation (24) and the sectoral RER instrument defined in equation (C.1). Additionally, column 4 uses the skill-specific-wage instruments defined in equation (C.2). In column 3, we exclude firms that import from any country to which they also export more than 10% of their total export value over the period. We report the minimum value of the Sanderson and Windmeijer (2016) first-stage F statistics obtained by running the equivalent 2SLS estimation. Robust standard errors are reported in parentheses.

In Table D.3 we estimate demand elasticities assuming a three-nest CES, where ε is the elasticity across firms within the most disaggregated 4-digit sector nest and η is the elasticity across 4-digit sectors within 1-digit sectors. All specifications are estimated using GMM. In the first column, we do not instrument (each covariate serves as its own instrument). Column 2 serves as our baseline specification with three nests. It instruments using firm- and sector-level RER instruments. Column 3 replicates column 2, but we exclude firms that import from any country to which they also export more than 10% of their total export value over the period (averaged across years). Finally, column 4 replicates column 2, but uses the two SSW instruments, in addition to the RER instrument.

Tables D.4, D.5, D.6, and D.7 revisit the estimation of the firm-level production function parameters.

In Table D.4, we estimate each individual equation (26) and (27) separately via 2SLS, rather than as a system via GMM. This exercise allows us to assess the strength of the first stage, which is strong. It also allows us to assess the importance of imposing cross-equation restrictions. While the estimating equations are identical to those in our baseline, this approach does not leverage the cross-equation restrictions. Taking the difference

Table D.4: Firm-Level Production Elasticities (CRESH): 2SLS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\Delta w_\ell - \Delta w_e$	1.15		1.04		1.14		1.00		0.93		1.00	
	(0.13)		(0.10)		(0.13)		(0.13)		(0.10)		(0.13)	
$\frac{\varepsilon}{\varepsilon-1}\Delta r - \Delta x_e$	-0.29	-0.21	-0.28	-0.16	-0.25	-0.18	-0.09	-0.09	-0.12	-0.10	-0.07	-0.08
	(0.07)	(0.06)	(0.05)	(0.05)	(0.06)	(0.05)	(0.10)	(0.08)	(0.07)	(0.06)	(0.08)	(0.07)
$\Delta w_h - \Delta w_e$		0.52		0.46		0.51		0.59		0.56		0.59
		(0.11)		(0.09)		(0.11)		(0.11)		(0.09)		(0.11)
Observations	101,087	101,087	100,998	100,998	101,087	101,087	68,377	68,377	68,318	68,318	68,377	68,377
Year-Sector FE	2 dig	2 dig	1 dig	1 dig	2 dig	2 dig	2 dig	2 dig	1 dig	1 dig	2 dig	2 dig
IV	*RER*	*RER*	*RER*	*RER*	*RER*	*RER*	RER^e	RER^e	RER^e	RER^e	RER^e	RER^e
ε	4.49	4.49	4.49	4.49	3.13	3.13	4.49	4.49	4.49	4.49	3.13	3.13
SW Min F-Stat	36.42	35.94	65.86	59.76	38.70	36.88	23.62	22.82	44.64	43.44	25.59	24.54

Notes: This table reports regression coefficients for the system of equations (26) and (27), with each equation estimated separately by 2SLS using the same set of instruments of Table 4. All columns include year-sector effects with sectors defined at the 2-digit level (columns 1 and 2, 5-8, and 11 and 12) or at the 1-digit level (columns 3-4 and 9-10). Robust standard errors are reported in parentheses. SW Min F-Stat is the minimum value of the Sanderson and Windmeijer (2016) first-stage F statistics.

between the estimated coefficients on $\Delta w_\ell - \Delta w_e$ and $\Delta w_h - \Delta w_e$ yields a slightly stronger measure of within-firm capital-skill complementarity.

In column 1 of Table D.5, we replace the RER instrument using a variant of our WES instrument defined in equation (C.3); in this variant, the shock $\Delta \log Exp_{ckt}$ is replaced with the change in the logarithm of the unit value of exports from country c in equipment product k between $t-5$ and t and the import shares, therefore, sum only across equipment products. In columns 2 and 3 we vary the value of ε, choosing its maximum and minimum (instrumented) values estimated in Table 3 at the 4-digit level. In column 4 we revisit our approach to dealing with outliers. In particular, we neither winsorize the dependent variables nor truncate the instruments.

Columns 1 and 3 of Table D.6 replicate the CXES estimation results in columns 1 and 2 of Table 5. Columns 2 and 4 provide robustness in which we use 1-digit instead of 2-digit sectors. Table D.7 estimates firm-level production elasticities in the CRESH system using the value of $\varepsilon = 5.19$ estimated assuming a three-level nested demand system.

Tables D.8 and D.9 provide additional quantitative results.

Table D.5: Firm-Level Production Elasticities (CRESH), Robustness

	(1)	(2)	(3)	(4)
σ_ℓ	1.45	1.07	1.03	1.16
	(0.21)	(0.16)	(0.16)	(0.16)
σ_h	1.22	0.87	0.89	0.99
	(0.25)	(0.13)	(0.13)	(0.14)
σ_e	1.30	1.35	1.25	1.47
	(0.20)	(0.28)	(0.24)	(0.27)
$\overline{\sigma}_{\ell e,i}$	1.43	1.33	1.22	1.42
	(0.20)	(0.25)	(0.22)	(0.23)
$\overline{\sigma}_{he,i}$	1.20	1.07	1.05	1.20
	(0.22)	(0.18)	(0.17)	(0.19)
$\overline{\sigma}_{\ell h,i}$	1.35	0.85	0.87	0.95
	(0.24)	(0.11)	(0.12)	(0.13)
Observations	72,754	101,087	101,087	103,080
Year-Sector FE	2 dig	2 dig	2 dig	2 dig
IV	*WESpe*	*RER*	*RER*	*RER*
Sel IV	Yes	Yes	Yes	No
Wins Dep Var	Yes	Yes	Yes	No
ε	4.49	5.44	3.37	4.49
$\Pr[\sigma_\ell < \sigma_h]$	0.06	0.03	0.05	0.10

Notes: This table reports regression coefficients for the system of equations (26) and (27) estimated using GMM for a given value of ε from Table 3, and with variables demeaned by year-sector means, with sectors defined at the 2-digit level. Each reported value of $\overline{\sigma}_{ff',i}$ is constructed by averaging across the firm-year observations used in the estimation sample for that column. All columns use the skill-specific-wage instruments defined in equation (C.2) and the world-export-supply instrument defined in equation (C.3) in Appendix C.3. Additionally, columns 2-4 use the firm-level RER instrument defined in equation (24), while column 1 uses the equipment-only-unit-value WES instrument defined in the text. Column 4 drops the winsorization of the dependent variables and the truncation of the WES and RER instruments. Bootstrap standard errors are reported in parentheses, obtained by bootstrapping the estimation of ε and of this system 200 times. $\Pr[\sigma_\ell < \sigma_h]$ reports the share of bootstrapped estimates in which σ_ℓ is lower than σ_h.

Table D.6: Firm-Level Production Elasticities, CXES

	(1)	(2)	(3)	(4)
$\zeta_{\ell e}$	**1.24**	1.02	0.96	0.89
	(0.24)	(0.16)	(0.17)	(0.11)
ζ_{he}	**1.06**	0.94	0.86	0.81
	(0.17)	(0.12)	(0.15)	(0.10)
$\zeta_{\ell h}$	**0.86**	0.76	0.87	0.78
	(0.13)	(0.09)	(0.16)	(0.09)
Observations	**101,087**	100,998	68,377	68,318
Year-Sector FE	**2 dig**	1 dig	2 dig	1 dig
IV	***RER***	*RER*	RER^e	RER^e
$\Pr[\zeta_{\ell e} < \zeta_{he}]$	**0.04**	0.13	0.17	0.09

Notes: This table replicates the CXES estimation results from Table 5 but includes two additional specifications in which we include one-digit sector x year fixed effects instead of two-digit sector x year fixed effects.

Table D.7: Firm-Level Production Elasticities (CRESH), 3 Nests

	(1)	(2)	(3)	(4)
σ_ℓ	**1.07**	0.89	0.92	0.84
	(0.16)	(0.11)	(0.14)	(0.09)
σ_h	**0.87**	0.79	0.79	0.74
	(0.11)	(0.09)	(0.12)	(0.08)
σ_e	**1.34**	1.10	0.93	0.88
	(0.26)	(0.17)	(0.19)	(0.11)
$\overline{\sigma}_{\ell e,i}$	**1.32**	1.05	0.97	0.90
	(0.23)	(0.15)	(0.17)	(0.11)
$\overline{\sigma}_{he,i}$	**1.07**	0.93	0.83	0.79
	(0.16)	(0.12)	(0.14)	(0.09)
$\overline{\sigma}_{\ell h,i}$	**0.85**	0.75	0.82	0.76
	(0.10)	(0.08)	(0.12)	(0.08)
Observations	**101,087**	100,998	68,377	68,318
Year-Sector FE	**2 dig**	1 dig	2 dig	1 dig
IV	***RER***	*RER*	RER^e	RER^e
$\Pr[\sigma_\ell < \sigma_h]$	**0.01**	0.06	0.07	0.05
ε	**5.19**	5.19	5.19	5.19

Notes: This table reports regression coefficients for the system of equations (26) and (27) estimated using GMM for a given value of ε from Table D.3, and with variables demeaned by year-sector means, with sectors defined at the 2-digit level or at the 1-digit level (column 2). Corresponding point estimates of σ_ℓ, σ_h, and σ_e are displayed. Each reported value of $\overline{\sigma}_{ff',i}$ is constructed by averaging across the firm-year observations used in the estimation sample for that column. Bootstrap standard errors are reported in parentheses, obtained by bootstrapping the estimation of ε and of this system 200 times and $\Pr[\sigma_\ell < \sigma_h]$ reports the share of bootstrapped estimates in which σ_ℓ is lower than σ_h.

Table D.8: Predicted Change in the Skill Premium, Uniform Shock, Back-of-the-envelope Calculation

	1997-2007	1990-2015
$\Delta \log W_e$	-0.23	-0.42
$\Delta \log W_\ell$	0.35	0.72
$\Delta \log W_e$ - $\Delta \log W_\ell$	-0.58	-1.14
$\Delta \log(W_h/W_\ell)$	0.04	0.09

Notes: This table reports the predicted log change in the skill premium $\Delta \log(W_h/W_\ell)$ in response to the observed fall in the equipment price relative to the low-skilled shadow wage. The predicted change is obtained by multiplying the total cumulated change in the relative equipment price by the average aggregate elasticities over the entire period from column 1 of Table 6. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low-skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labor.

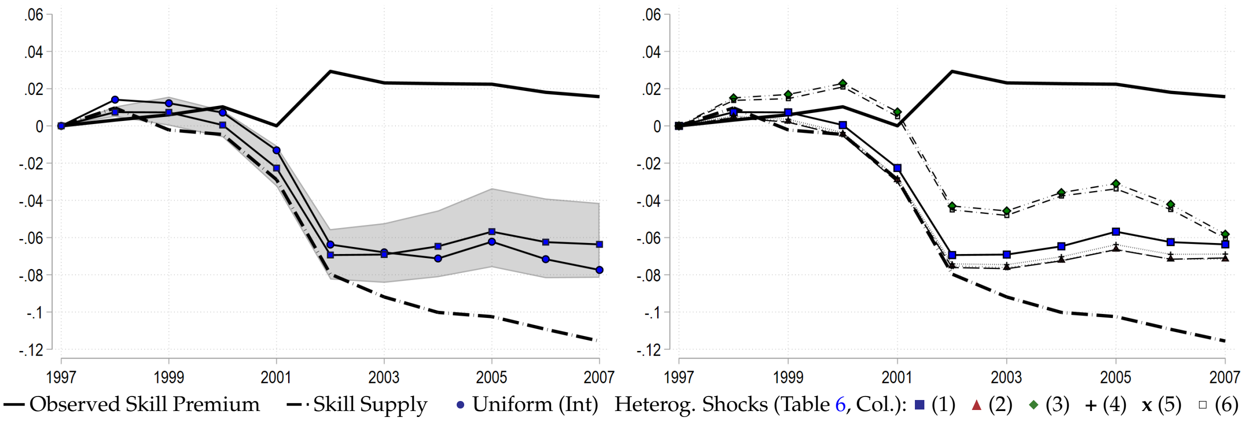
Table D.9: Decomposition of the Predicted Change in the Skill Premium

	Uniform Shock	Heterogeneous Shock
	$\Delta \log(W_h/W_\ell)$	$\Delta \log(W_h/W_\ell)$
Equipment	0.04	0.05
	[0.02, 0.06]	[0.04, 0.07]
Within firm complementarity	0.03	0.03
	[0.01, 0.06]	[0.00, 0.06]
Within firm substitution	-0.02	-0.02
	[-0.03, -0.01]	[-0.03, -0.01]
Cross firm	0.02	0.04
	[0.02, 0.02]	[0.03, 0.04]

Notes: This table reports the decomposition of the predicted log change in the skill premium in response to the observed fall in the equipment price relative to the low-skilled shadow wage. The left panel presents the results obtained using a uniform shock computed by aggregating the firm-level heterogeneous price shocks (see Section C.1.7). The right panel reports the estimates obtained using heterogeneous price shocks, analyzed in Section 5.2. The decomposition for the uniform shock is not exact due to rounding error. The table displays 90% confidence intervals obtained by bootstrapping the estimation of each σ_f 200 times.

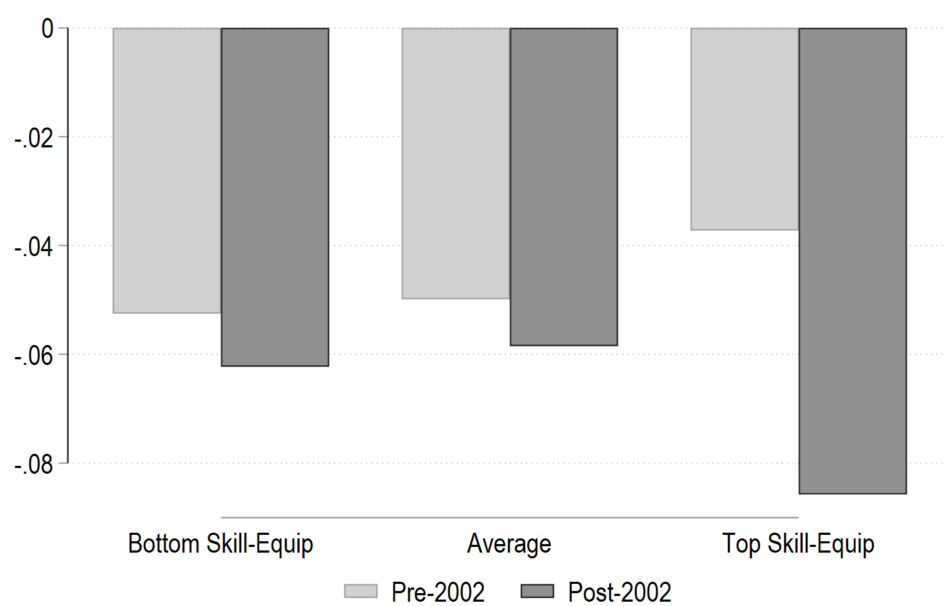
Figure D.1: Predicted vs Observed Skill Premium, Heterogeneous Shocks, CRESH

(a) Uniform vs Heterogeneous Shocks (b) Alternative Estimates



Notes: Panel (a) plots the evolution of the predicted skill premium ($d\log(W_h/W_\ell)$) in response to firm-specific relative equipment prices (see Section A.3.2). We proxy the aggregate change in the low-skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labor. The figure displays a 90% confidence interval obtained by bootstrapping the estimation of each σ_f 200 times. We compare the results against the evolution of the predicted skill premium in response to a uniform change in relative equipment price computed by aggregating the firm-specific equipment price changes (see Section C.1.7). Panel (b) plots the variation in the predicted skill premium across the different estimated aggregate elasticities of Table 6.

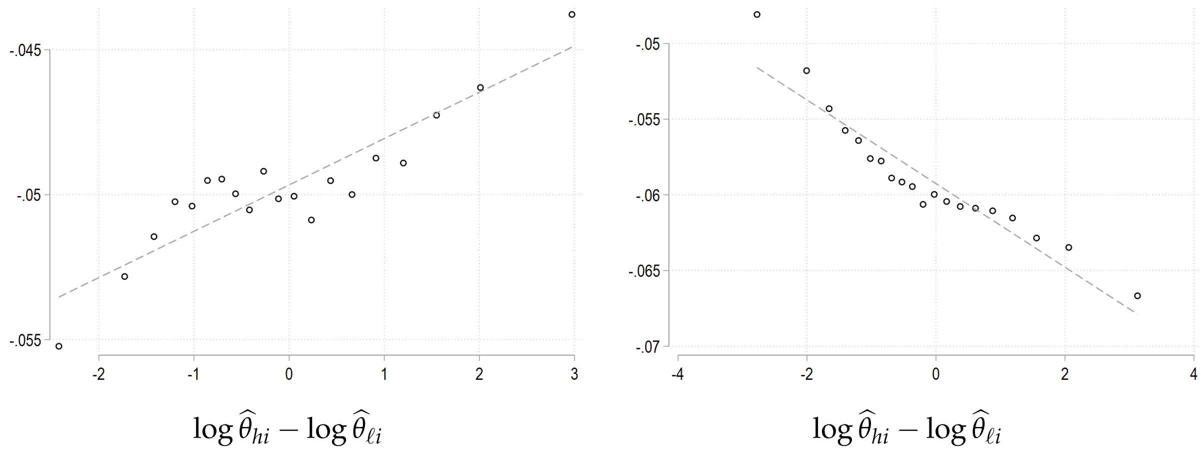
Figure D.2: Average Annual Equipment Shocks by Skill and Equipment Intensity



Notes: The chart reports the average firm-level equipment price shock by bins of skill and equipment intensity, $\Lambda_i\left(\widehat{\theta}_{hi} - \widehat{\theta}_{\ell i}\right)\theta_{ei}$, and further split into the pre- and post-2002 periods. The bins are defined on the average skill and equipment intensity over the entire period, and the top and bottom bins represent 1% of firms (around 7000 firms each).

Figure D.3: Equipment Shocks by Skill Intensity

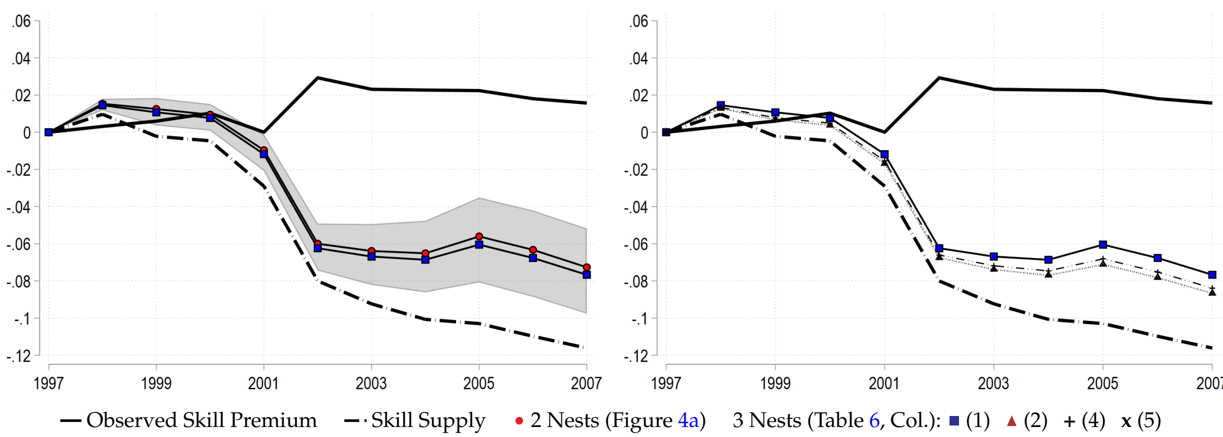
(a) Pre 2002 (b) Post 2002



Notes: The figure plots a binscatter of the annual change in firm-specific equipment prices over the log of the skill intensity ratio $(\widehat{\theta}_{hi}/\widehat{\theta}_{\ell i})$.

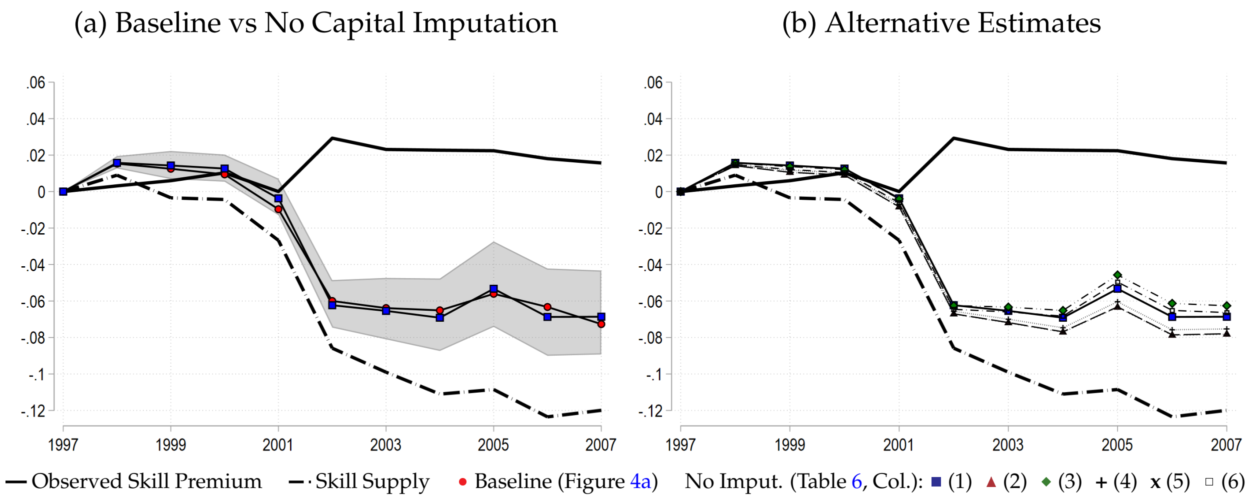
Figure D.4: Predicted Skill Premium, Uniform Shock, 3-nest Aggregation

(a) 3- vs 2-Nest Aggregation (b) Alternative Estimates



Notes: Panel (a) plots the evolution of the predicted skill premium $(d\log(W_h/W_\ell))$ in response to the observed fall in the relative equipment price and the observed change in the skill supply. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low-skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labor (see Section C.1.7). The figure displays a 90% confidence interval obtained by bootstrapping the estimation of each σ_f 200 times. Panel (b) plots the variation in the predicted skill premium across the different estimated aggregate elasticities of Table 6, excluding columns 3 and 6 in which we set $\eta = 0.5$.

Figure D.5: Predicted Skill Premium, Uniform Shock, No Capital Imputation



Notes: Panel (a) plots the evolution of the predicted skill premium ($d\log(W_h/W_\ell)$) in response to the observed fall in the relative equipment price and the observed change in the skill supply. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low-skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labor (see Section C.1.7). The figure displays a 90% confidence interval obtained by bootstrapping the estimation of each σ_f 200 times. Panel (b) plots the variation in the predicted skill premium across the different estimated aggregate elasticities of Table 6.

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